FUNCTIONS

8 TH GRADE

Lesson 5: Graphs of Functions and Equations (from Module 5)

Classwork

Exercises 1–3

1. The distance that Giselle can run is a function of the amount of time she spends running. Giselle runs 3 miles in 21 minutes. Assume she runs at a constant rate.

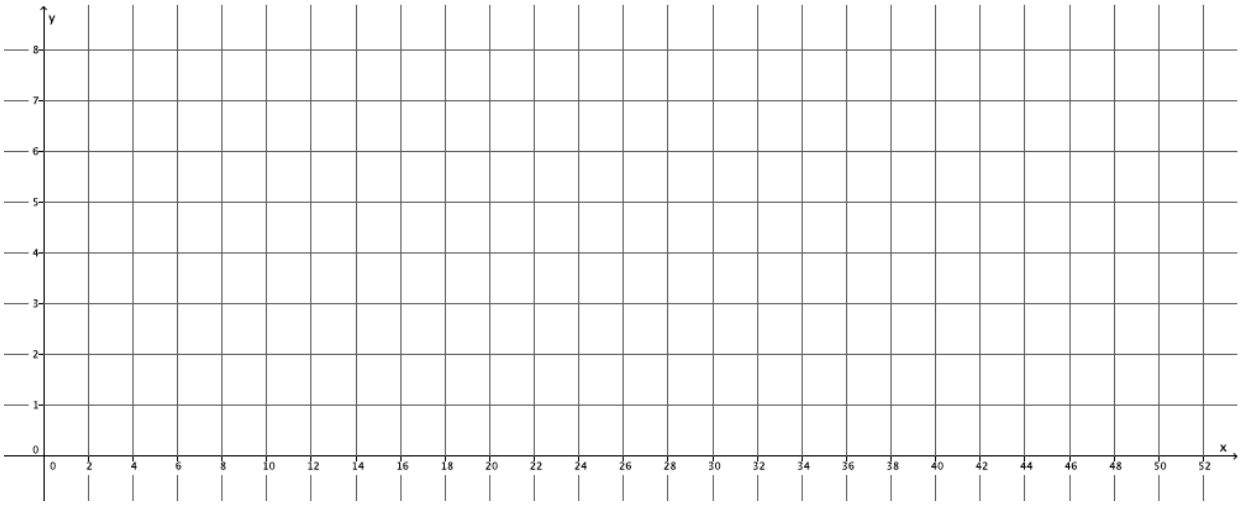
   a. Write an equation in two variables that represents her distance run, y, as a function of the time, x, she spends running.

   b. Use the equation you wrote in part (a) to determine how many miles Giselle can run in 14 minutes.

   c. Use the equation you wrote in part (a) to determine how many miles Giselle can run in 28 minutes.

   d. Use the equation you wrote in part (a) to determine how many miles Giselle can run in 7 minutes.

e. The input of the function, x, is time, and the output of the function, y, is the distance Giselle ran. Write the inputs and outputs from parts (b)–(d) as ordered pairs, and plot them as points on a coordinate plane.



f. What shape does the graph of the points appear to take?

g. Is the function continuous or discrete?

h. Use the equation you wrote in part (a) to determine how many miles Giselle can run in 36 minutes. Write your answer as an ordered pair, as you did in part (e), and include the point on the graph. Is the point in a place where you expected it to be? Explain.

i. Assume you used the rule that describes the function to determine how many miles Giselle can run for any given time and wrote each answer as an ordered pair. Where do you think these points would appear on the graph?

j. What do you think the graph of this function will look like? Explain.

k. Connect the points you have graphed to make a line. Select a point on the graph that has integer coordinates. Verify that this point has an output that the function would assign to the input.

l. Sketch the graph of the equation $y = \frac{1}{7}x$ using the same coordinate plane in part (e). What do you notice about the graph of the function that describes Giselle's constant rate of running and the graph of the equation $y = \frac{1}{7}x$?

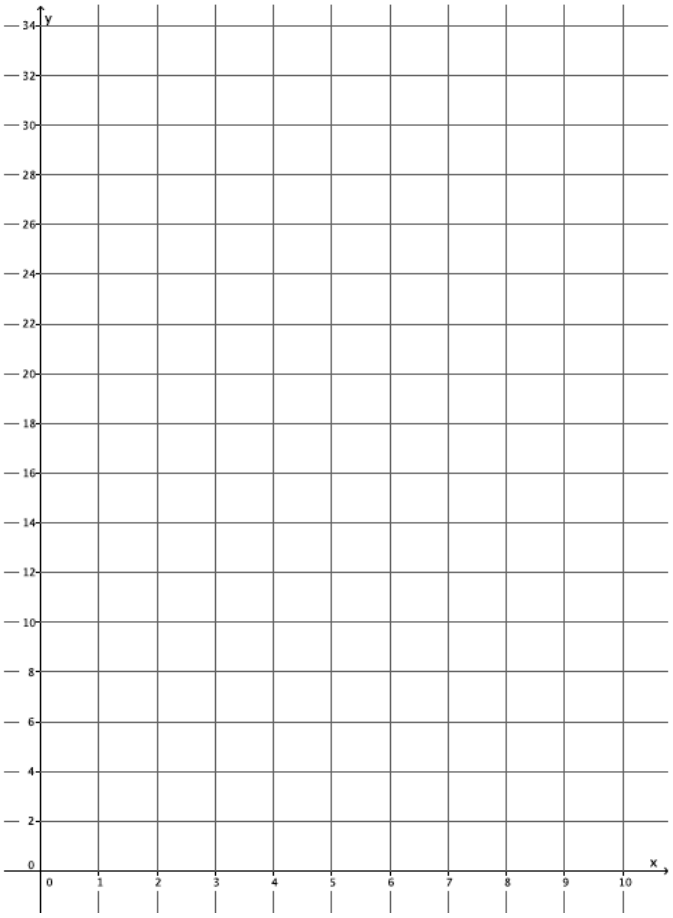
2. Sketch the graph of the equation $y = x^2$ for positive values of x. Organize your work using the table below, and then answer the questions that follow.

x	y
0	
1	
2	
3	
4	
5	
6	

a. Plot the ordered pairs on the coordinate plane.

b. What shape does the graph of the points appear to take?

c. Is this equation a linear equation? Explain.



d. An area function A for a square with length of a side s has the rule so that it assigns to each input an output, the area of the square, A. Write the rule for this function.

e. What do you think the graph of this function will look like? Explain.

f. Use the function you wrote in part (d) to determine the area of a square with side length 2.5. Write the input and output as an ordered pair. Does this point appear to belong to the graph of $y = x^2$?

3. The number of devices a particular manufacturing company can produce is a function of the number of hours spent making the devices. On average, 4 devices are produced each hour. Assume that devices are produced at a constant rate.

   a. Write an equation in two variables that represents the number of devices, y, as a function of the time the company spends making the devices, x.

   b. Use the equation you wrote in part (a) to determine how many devices are produced in 8 hours.

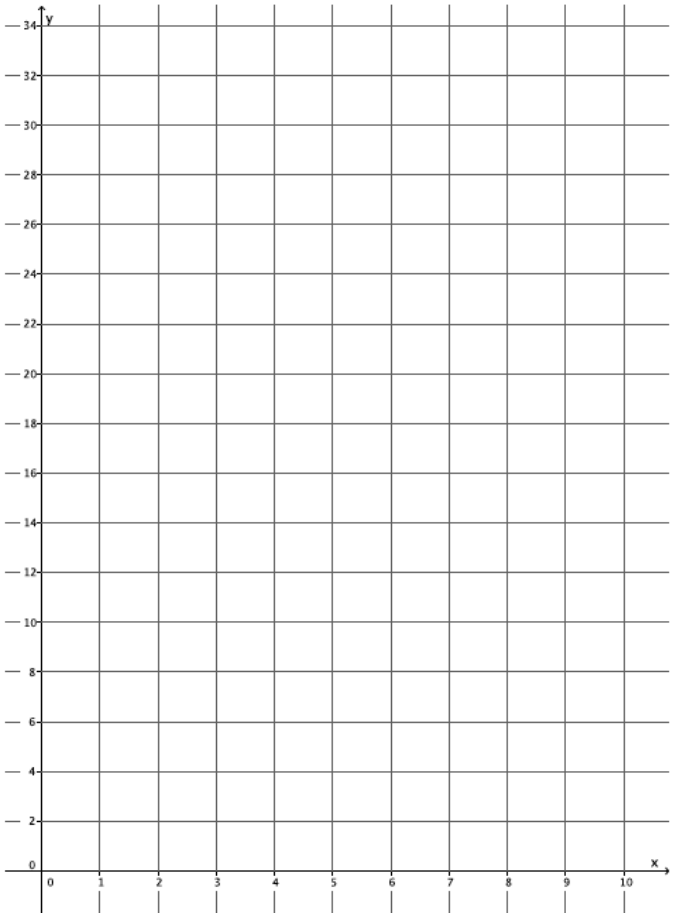
c. Use the equation you wrote in part (a) to determine how many devices are produced in 6 hours.

d. Use the equation you wrote in part (a) to determine how many devices are produced in 4 hours.

e. The input of the function, x, is time, and the output of the function, y, is the number of devices produced. Write the inputs and outputs from parts (b)–(d) as ordered pairs, and plot them as points on a coordinate plane.

f. What shape does the graph of the points appear to take?

g. Is the function continuous or discrete?



h. Use the equation you wrote in part (a) to determine how many devices are produced in 1.5 hours. Write your answer as an ordered pair, as you did in part (e), and include the point on the graph. Is the point in a place where you expected it to be? Explain.

i. Assume you used the rule that describes the function to determine how many devices are produced for any given time and wrote each answer as an ordered pair. Where do you think these points would appear on the graph?

j. What do you think the graph of this function will look like? Explain.

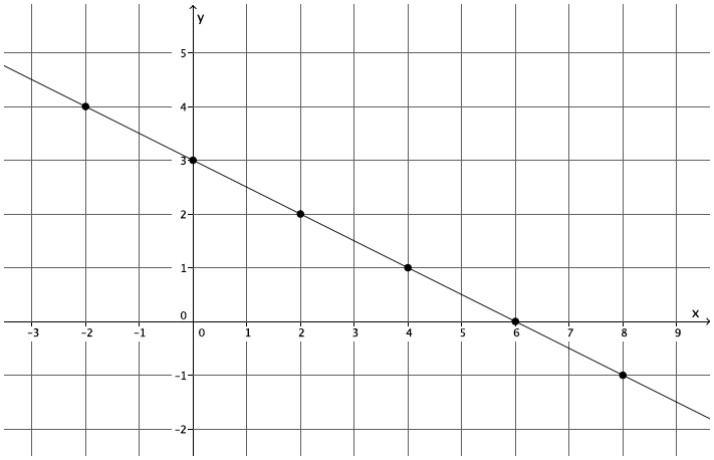
k. Connect the points you have graphed to make a line. Select a point on the graph that has integer coordinates. Verify that this point has an output that the function would assign to the input.

l. Sketch the graph of the equation $y = 4x$ using the same coordinate plane in part (e). What do you notice about the graph of the function that describes the company's constant rate of producing devices and the graph of the equation $y = 4x$?

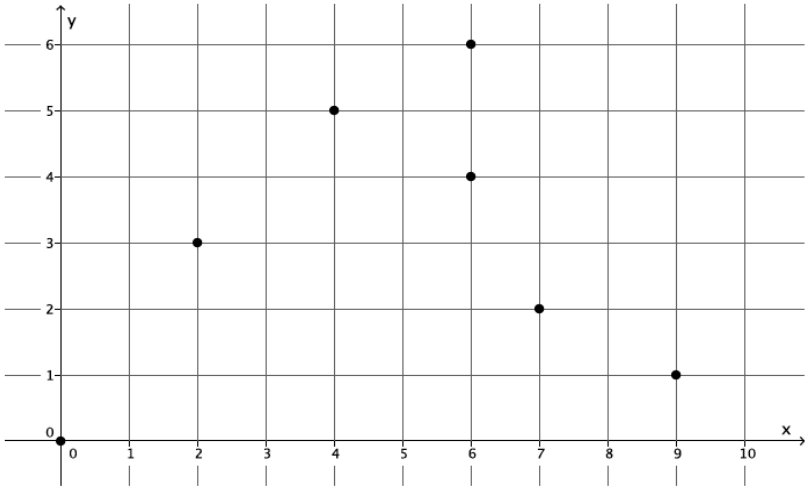
Exploratory Challenge/Exercise 4

4. Examine the three graphs below. Which, if any, could represent the graph of a function? Explain why or why not for each graph.

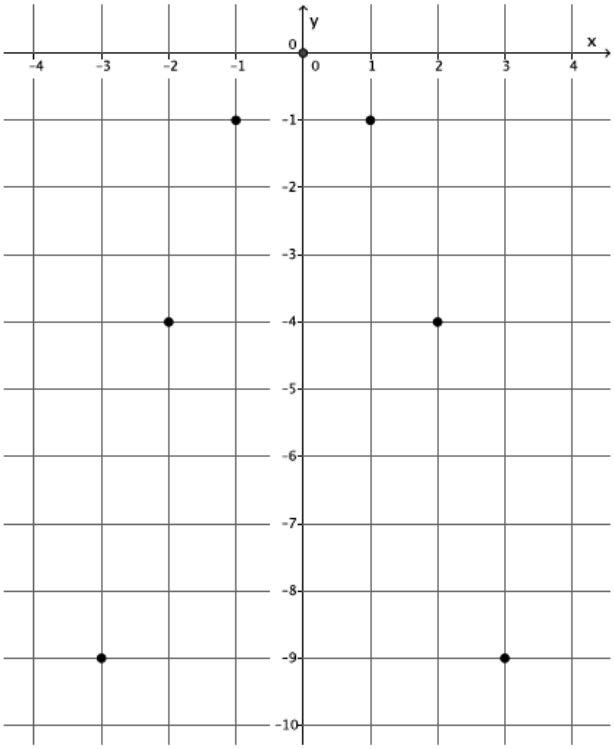
Graph 1:



Graph 2:



Graph 3:



Lesson 6: Graphs of Linear Functions and Rate of Change (from Module 5)

Classwork

Opening Exercise

Functions 1, 2, and 3 have the tables shown below. Examine each of them, make a conjecture about which will be linear, and justify your claim.

Input	Output
2	5
4	7
5	8
8	11

Input	Output
2	4
3	9
4	16
5	25

Input	Output
0	-3
1	1
2	6
3	9

Exercise

A function assigns the inputs and corresponding outputs shown in the table below.

Input	Output
1	2
2	−1
4	−7
6	−13

a. Is the function a linear function? Check at least three pairs of inputs and their corresponding outputs.

b. What equation describes the function?

c. What will the graph of the function look like? Explain.

Lesson 7: Comparing Linear Functions and Graphs (from Module 5)

Classwork

Exercises

Exercises 1–4 provide information about functions. Use that information to help you compare the functions and answer the questions.

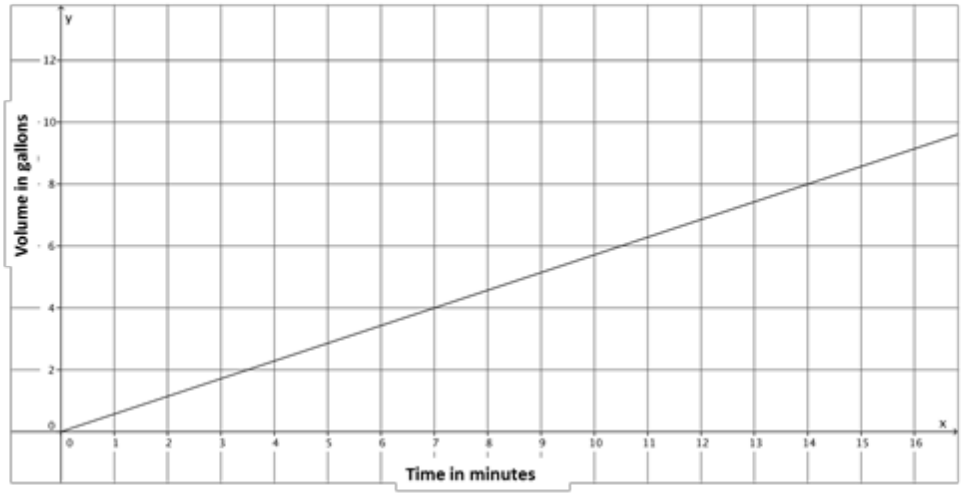
1. Alan and Margot drive from City A to City B, a distance of 147 miles. They take the same route and drive at constant speeds. Alan begins driving at 1:40 p.m. and arrives at City B at 4:15 p.m. Margot's trip from City A to City B can be described with the equation $y = 64x$, where y is the distance traveled and x is the time in minutes spent traveling. Who gets from City A to City B faster?

2. You have recently begun researching phone billing plans. Phone Company A charges a flat rate of $\$75$ a month. A flat rate means that your bill will be $\$75$ each month with no additional costs. The billing plan for Phone Company B is a linear function of the number of texts that you send that month. That is, the total cost of the bill changes each month depending on how many texts you send. The table below represents the inputs and the corresponding outputs that the function assigns.

Input (number of texts)	Output (cost of bill)
50	$\$50$
150	$\$60$
200	$\$65$
500	$\$95$

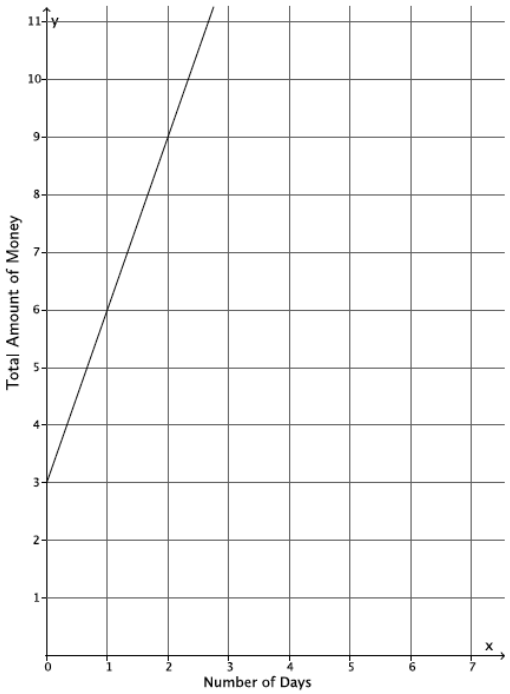
At what number of texts would the bill from each phone plan be the same? At what number of texts is Phone Company A the better choice? At what number of texts is Phone Company B the better choice?

3. A function describes the volume of water, y, that flows from Faucet A in gallons for x minutes. The graph below is the graph of this linear function. Faucet B's water flow can be described by the equation $y = \frac{5}{6}x$, where y is the volume of water in gallons that flows from the faucet in x minutes. Assume the flow of water from each faucet is constant. Which faucet has a faster rate of flow of water? Each faucet is being used to fill tubs with a volume of 50 gallons. How long will it take each faucet to fill the tub? How do you know? The tub that is filled by Faucet A already has 15 gallons in it. If both faucets are turned on at the same time, which faucet will fill its tub faster?



4. Two people, Adam and Bianca, are competing to see who can save the most money in one month. Use the table and the graph below to determine who will save more money at the end of the month. State how much money each person had at the start of the competition.

Adam's Savings:



Bianca's Savings:

Input (Number of Days)	Output (Total amount of money)
5	$17
8	$26
12	$38
20	$62

Lesson 8: Graphs of Simple Nonlinear Functions (from Module 5)

Classwork

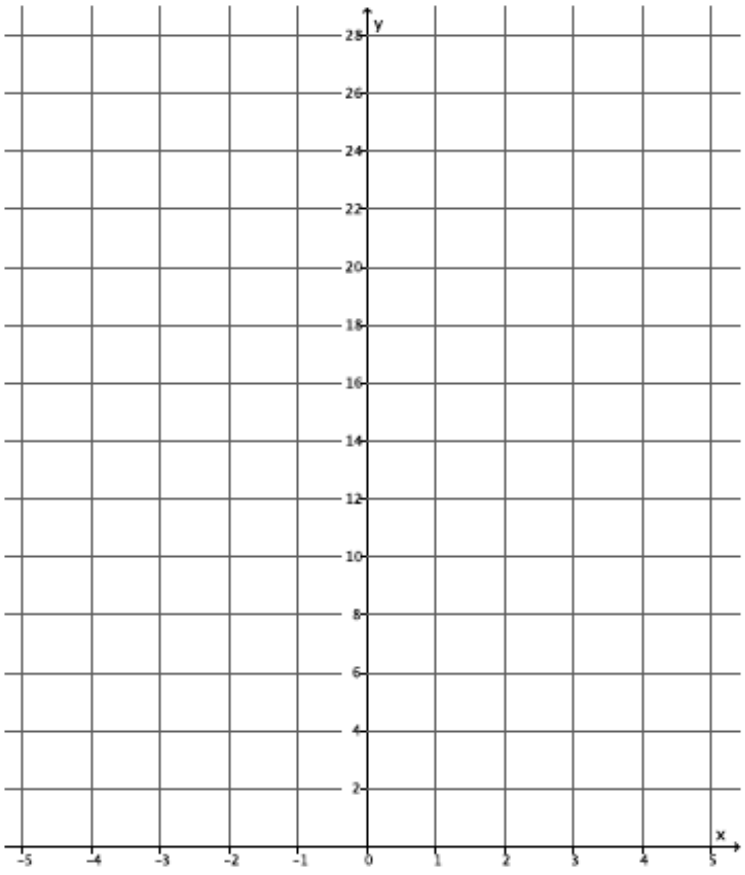
Exercises

1. A function has the rule so that each input of x is assigned an output of x^2.
   a. Do you think the function is linear or nonlinear? Explain.

   b. Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.

Input (x)	Output (x^2)
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

   c. Plot the inputs and outputs as points on the coordinate plane where the output is the y-coordinate.



d. What shape does the graph of the points appear to take?

e. Find the rate of change using rows 1 and 2 from the table above.

f. Find the rate of change using rows 2 and 3 from the above table.

g. Find the rate of change using any two other rows from the above table.

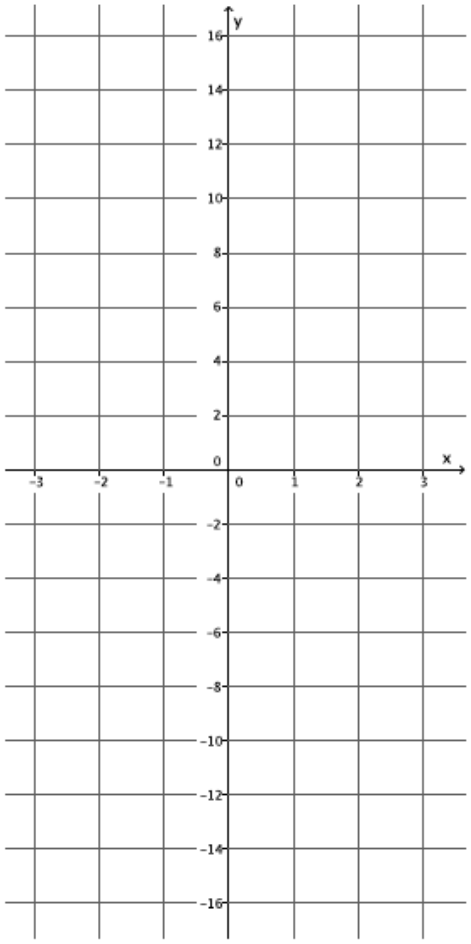
h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

2. A function has the rule so that each input of x is assigned an output of x^3.

   a. Do you think the function is linear or nonlinear? Explain.

   b. Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.

Input (x)	Output (x^3)
-2.5	
-2	
-1.5	
-1	
-0.5	
0	
0.5	
1	
1.5	
2	
2.5	



   c. Plot the inputs and outputs as points on the coordinate plane where the output is the y-coordinate.

   d. What shape does the graph of the points appear to take?

   e. Find the rate of change using rows 2 and 3 from the table above.

f. Find the rate of change using rows 3 and 4 from the table above.

g. Find the rate of change using rows 8 and 9 from the table above.

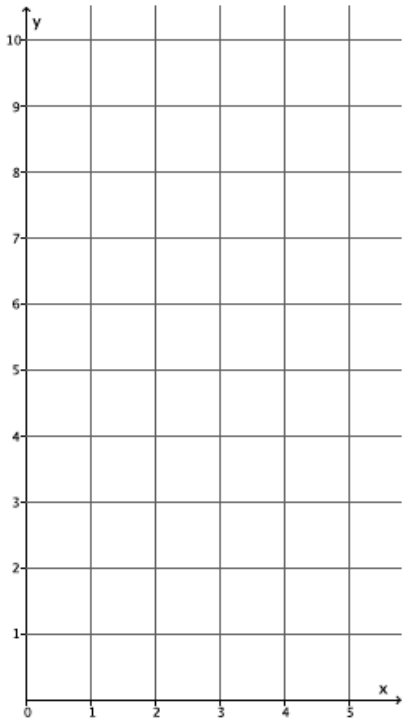
h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

3. A function has the rule so that each input of x is assigned an output of $\frac{1}{x}$ for values of $x > 0$.

   a. Do you think the function is linear or nonlinear? Explain.

   b. Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.

Input (x)	Output $\left(\frac{1}{x}\right)$
0.1	
0.2	
0.4	
0.5	
0.8	
1	
1.6	
2	
2.5	
4	
5	



   c. Plot the inputs and outputs as points on the coordinate plane where the output is the y-coordinate.

   d. What shape does the graph of the points appear to take?

   e. Find the rate of change using rows 1 and 2 from the table above.

   f. Find the rate of change using rows 2 and 3 from the table above.

g. Find the rate of change using any two other rows from the table above.

h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

In Exercises 4–10, the rule that describes a function is given. If necessary, use a table to organize pairs of inputs and outputs, and then plot each on a coordinate plane to help answer the questions.

4. What shape do you expect the graph of the function described by $y = x$ to take? Is it a linear or nonlinear function?

5. What shape do you expect the graph of the function described by $y = 2x^2 - x$ to take? Is it a linear or nonlinear function?

6. What shape do you expect the graph of the function described by $3x + 7y = 8$ to take? Is it a linear or nonlinear function?

7. What shape do you expect the graph of the function described by $y = 4x^3$ to take? Is it a linear or nonlinear function?

8. What shape do you expect the graph of the function described by $\frac{3}{x} = y$ to take? Is it a linear or nonlinear function?

9. What shape do you expect the graph of the function described by $\frac{4}{x^2} = y$ to take? Is it a linear or nonlinear function?

10. What shape do you expect the graph of the equation $x^2 + y^2 = 36$ to take? Is it a linear or nonlinear? Is it a function? Explain.

Lesson 1: Modeling Linear Relationships

Classwork

Example 1: Logging On

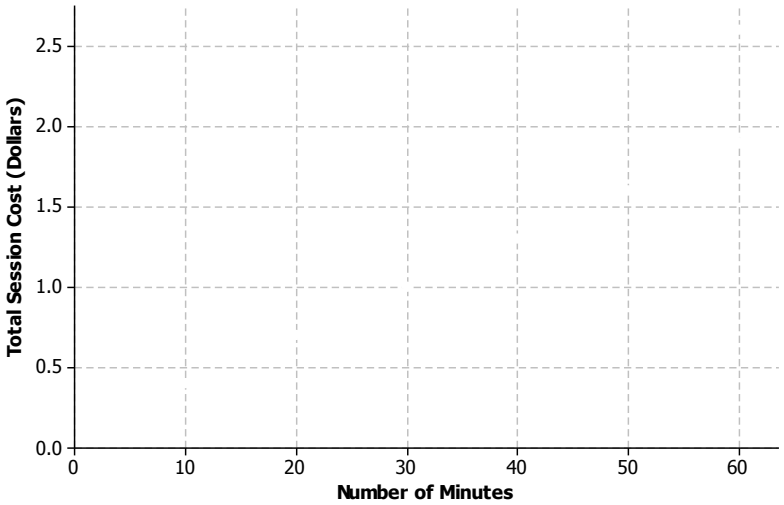
Lenore has just purchased a tablet computer, and she is considering purchasing an internet access plan so that she can connect to the Internet wirelessly from virtually anywhere in the world. One company offers an internet access plan so that when a person connects to the company's wireless network, the person is charged a fixed access fee for connecting, PLUS an amount for the number of minutes connected based upon a constant usage rate in dollars per minute.

Lenore is considering this company's plan, but the company's advertisement does not state how much the fixed access fee for connecting is, nor does it state the usage rate. However, the company's website says that a 10-minute session costs \$0.40, a 20-minute session costs \$0.70, and a 30-minute session costs \$1.00. Lenore decides that she will use these pieces of information to determine both the fixed access fee for connecting and the usage rate.

Exercises 1–6

1. Lenore makes a table of this information and a graph where *number of minutes* is represented by the horizontal axis and *total session cost* is represented by the vertical axis. Plot the three given points on the graph. These three points appear to lie on a line. What information about the access plan suggests that the correct model is indeed a linear relationship?

Number of Minutes	Total Session Cost
0	
10	\$0.40
20	\$0.70
30	\$1.00
40	
50	
60	



2. The rate of change describes how the total cost changes with respect to time.
   a. When the number of minutes increases by 10 (such as from 10 minutes to 20 minutes or from 20 minutes to 30 minutes), how much does the charge increase?

   b. Another way to say this would be the "usage charge per 10 minutes of use." Use that information to determine the increase in cost based on only 1 minute of additional usage. In other words, find the "usage charge per minute of use."

3. The company's pricing plan states that the usage rate is constant for any number of minutes connected to the Internet. In other words, the increase in cost for 10 more minutes of use (the value that you calculated above) will be the same whether you increase from 20 to 30 minutes, 30 to 40 minutes, etc. Using this information, determine the total cost for 40 minutes, 50 minutes, and 60 minutes of use. Record those values in the table, and plot the corresponding points on the graph in Exercise 1.

4. Using the table and the graph in Exercise 1, compute the hypothetical cost for 0 minutes of use. What does that value represent in the context of the values that Lenore is trying to figure out?

5. On the graph in Exercise 1, draw a line through the points representing 0 to 60 minutes of use under this company's plan. The slope of this line is equal to the rate of change, which in this case is the usage rate.

6. Using x for the number of minutes and y for total cost in dollars, write a function to model the linear relationship between minutes of use and total cost.

Example 2: Another Rate Plan

A second wireless access company has a similar method for computing its costs. Unlike the first company that Lenore was considering, this second company explicitly states its access fee is $\$0.15$, and its usage rate is $\$0.04$ per minute.

$$\text{Total Session Cost} = \$0.15 + \$0.04(\text{number of minutes})$$

Exercises 7–16

7. Let x represent the number of minutes used and y represent the total session cost. Construct a linear function that models the total session cost based on the number of minutes used.

8. Using the linear function constructed in Exercise 7, determine the total session cost for sessions of 0, 10, 20, 30, 40, 50, and 60 minutes, and fill in these values in the table below.

Number of Minutes	Total Session Cost
0	
10	
20	
30	
40	
50	
60	

9. Plot these points on the original graph in Exercise 1, and draw a line through these points. In what ways does the line that represents this second company's access plan differ from the line that represented the first company's access plan?

MP3 download sites are a popular forum for selling music. Different sites offer pricing that depend on whether or not you want to purchase an entire album or individual songs "à la carte." One site offers MP3 downloads of individual songs with the following price structure: a $\$3$ fixed fee for monthly subscription PLUS a charge of $\$0.25$ per song.

10. Using x for the number of songs downloaded and y for the total monthly cost, construct a linear function to model the relationship between the number of songs downloaded and the total monthly cost.

11. Construct a table to record the total monthly cost (in dollars) for MP3 downloads of 10 songs, 20 songs, and so on up to 100 songs.

12. Plot the 10 data points in the table on a coordinate plane. Let the x-axis represent the number of songs downloaded and the y-axis represent the total monthly cost (in dollars) for MP3 downloads.

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid $\$200$. If 70 tickets are sold, the band will be paid $\$260$.

13. Determine the rate of change.

14. Let x represent the number of tickets sold and y represent the amount the band will be paid. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

15. What is the fee the band will be paid for playing the concert (not including ticket sales)?

16. How much will the band receive for each ticket sold?

Lesson 2: Interpreting Rate of Change and Initial Value

Classwork

Linear functions are defined by the equation of a line. The graphs and the equations of the lines are important for understanding the relationship between the two variables represented in the following example as x and y.

Example 1: Rate of Change and Initial Value

The equation of a line can be interpreted as defining a linear function. The graphs and the equations of lines are important in understanding the relationship between two types of quantities (represented in the following examples by x and y).

In a previous lesson, you encountered an MP3 download site that offers downloads of individual songs with the following price structure: a $\$3$ fixed fee for monthly subscription PLUS a fee of $\$0.25$ per song. The linear function that models the relationship between the number of songs downloaded and the total monthly cost of downloading songs can be written as

$$y = 0.25x + 3,$$

where x represents the number of songs downloaded, and y represents the total monthly cost (in dollars) for MP3 downloads.

a. In your own words, explain the meaning of 0.25 within the context of the problem.

b. In your own words, explain the meaning of 3 within the context of the problem.

The values represented in the function can be interpreted in the following way:

$$y = \underbrace{0.25x}_{\text{rate of change}} + \underbrace{3}_{\text{initial value}}$$

The coefficient of x is referred to as the rate of change. It can be interpreted as the change in the values of y for every one-unit increase in the values of x. When the rate of change is positive, the linear function is *increasing.* **In other words, *increasing* indicates that as the x-value increases, so does the y-value.** When the rate of change is negative, the linear function is *decreasing.* ***Decreasing*** **indicates that as the x-value increases, the y-value decreases.**	The constant value is referred to as the *initial value* or y-intercept and can be interpreted as the value of y when $x = 0$.

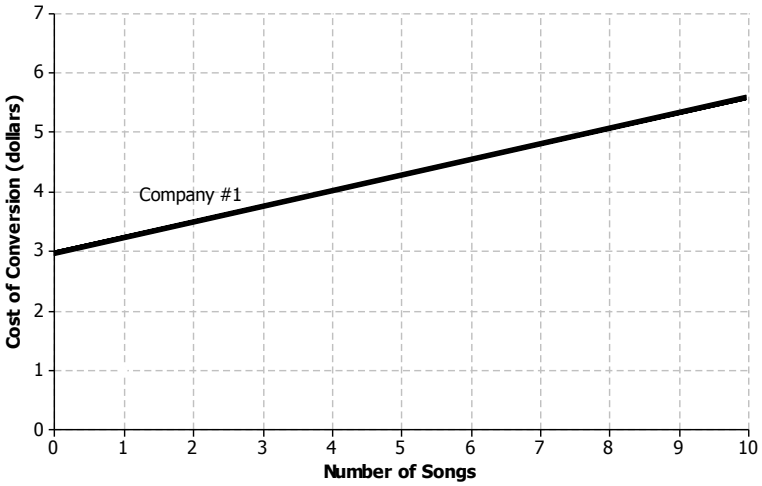
Exercises 1–6: Is It a Better Deal?

Another site offers MP3 downloads with a different price structure: a $\$2$ fixed fee for monthly subscription PLUS a fee of $\$0.40$ per song.

1. Write a linear function to model the relationship between the number of songs downloaded and the total monthly cost. As before, let x represent the number of songs downloaded and y represent the total monthly cost (in dollars) of downloading songs.

2. Determine the cost of downloading 0 songs and 10 songs from this site.

3. The graph below already shows the linear model for the first subscription site (Company 1): $y = 0.25x + 3$. Graph the equation of the line for the second subscription site (Company 2) by marking the two points from your work above (for 0 songs and 10 songs) and drawing a line through those two points.



4. Which line has a steeper slope? Which company's model has the more expensive cost per song?

5. Which function has the greater initial value?

6. Which subscription site would you choose if you only wanted to download 5 songs per month? Which company would you choose if you wanted to download 10 songs? Explain your reasoning.

Exercises 7–9: Aging Autos

7. When someone purchases a new car and begins to drive it, the mileage (meaning the number of miles the car has traveled) immediately increases. Let x represent the number of years since the car was purchased and y represent the total miles traveled. The linear function that models the relationship between the number of years since purchase and the total miles traveled is $y = 15000x$.
   a. Identify and interpret the rate of change.

   b. Identify and interpret the initial value.

   c. Is the mileage increasing or decreasing each year according to the model? Explain your reasoning.

8. When someone purchases a new car and begins to drive it, generally speaking, the resale value of the car (in dollars) goes down each year. Let x represent the number of years since purchase and y represent the resale value of the car (in dollars). The linear function that models the resale value based on the number of years since purchase is
$y = 20000 - 1200x$.

   a. Identify and interpret the rate of change.

   b. Identify and interpret the initial value.

   c. Is the resale value increasing or decreasing each year according to the model? Explain.

9. Suppose you are given the linear function $y = 2.5x + 10$.

   a. Write a story that can be modeled by the given linear function.

   b. What is the rate of change? Explain its meaning with respect to your story.

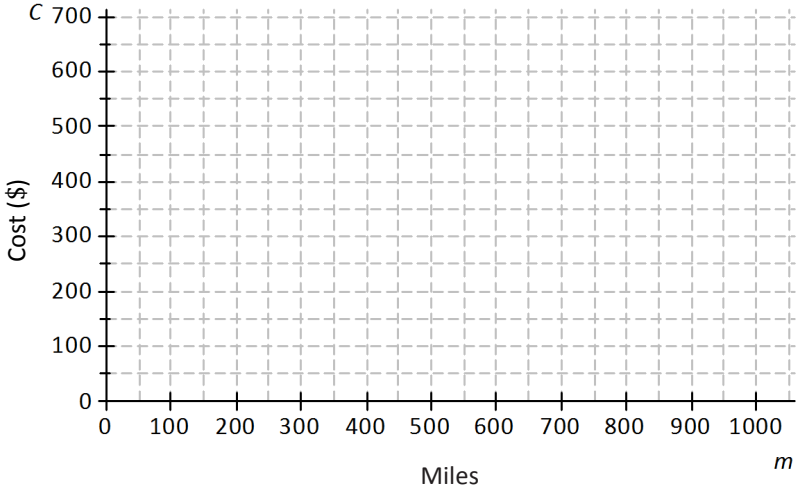
   c. What is the initial value? Explain its meaning with respect to your story.

Lesson 3: Representations of a Line

Classwork

Example 1: Rate of Change and Initial Value Given in the Context of the Problem

A truck rental company charges a $\$150$ rental fee in addition to a charge of $\$0.50$ per mile driven. In this problem, you will graph the linear function relating the total cost of the rental in dollars, C, to the number of miles driven, m, on the axes below.



a. If the truck is driven 0 miles, what will be the cost to the customer? How will this be shown on the graph?

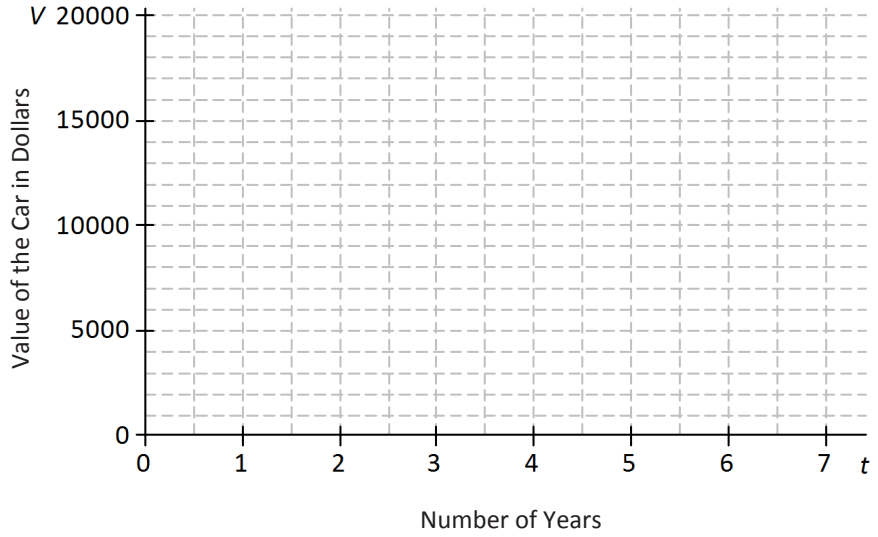
b. What is the rate of change that relates cost to number of miles driven? Explain what it means within the context of the problem.

c. On the axes given, sketch the graph of the linear function that relates C to m.

d. Write the equation of the linear function that models the relationship between number of miles driven and total rental cost.

Exercises

Jenna bought a used car for $\$18{,}000$. She has been told that the value of the car is likely to decrease by $\$2{,}500$ for each year that she owns the car. Let the value of the car in dollars be V and the number of years Jenna has owned the car be t.



1. What is the value of the car when $t = 0$? Show this point on the graph.

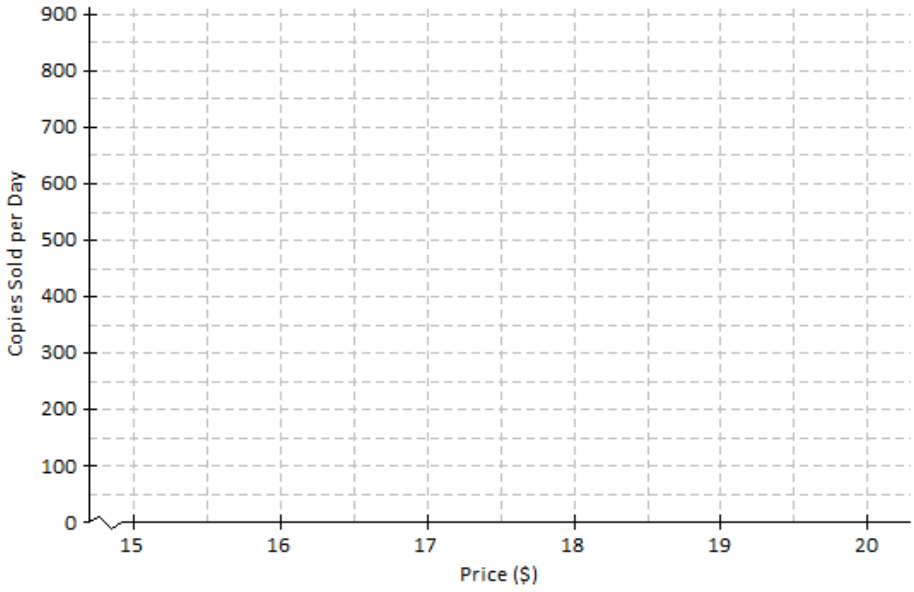
2. What is the rate of change that relates V to t? (Hint: Is it positive or negative? How can you tell?)

3. Find the value of the car when
   a. $t = 1$.
   b. $t = 2$.
   c. $t = 7$.

4. Plot the points for the values you found in Exercise 3, and draw the line (using a straight-edge) that passes through those points.

5. Write the linear function that models the relationship between the number of years Jenna has owned the car and the value of the car.

An online bookseller has a new book in print. The company estimates that if the book is priced at $\$15$, then 800 copies of the book will be sold per day, and if the book is priced at $\$20$, then 550 copies of the book will be sold per day.



6. Identify the ordered pairs given in the problem. Then, plot both on the graph.

7. Assume that the relationship between the number of books sold and the price is linear. (In other words, assume that the graph is a straight line.) Using a straight-edge, draw the line that passes through the two points.

8. What is the rate of change relating number of copies sold to price?

9. Based on the graph, if the company prices the book at $\$18$, about how many copies of the book can they expect to sell per day?

10. Based on the graph, approximately what price should the company charge in order to sell 700 copies of the book per day?

Lesson 4: Increasing and Decreasing Functions

Classwork

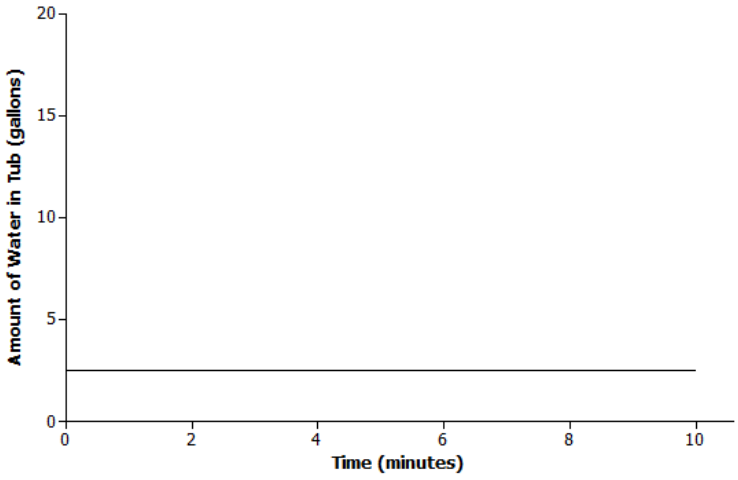
Graphs are useful tools in terms of representing data. They provide a visual story, highlighting important facts that surround the relationship between quantities.

The graph of a linear function is a line. The slope of the line can provide useful information about the functional relationship between the two types of quantities:

- A linear function whose graph has a positive slope is said to be an *increasing function.*
- A linear function whose graph has a negative slope is said to be a *decreasing function.*
- A linear function whose graph has a zero slope is said to be a *constant function.*

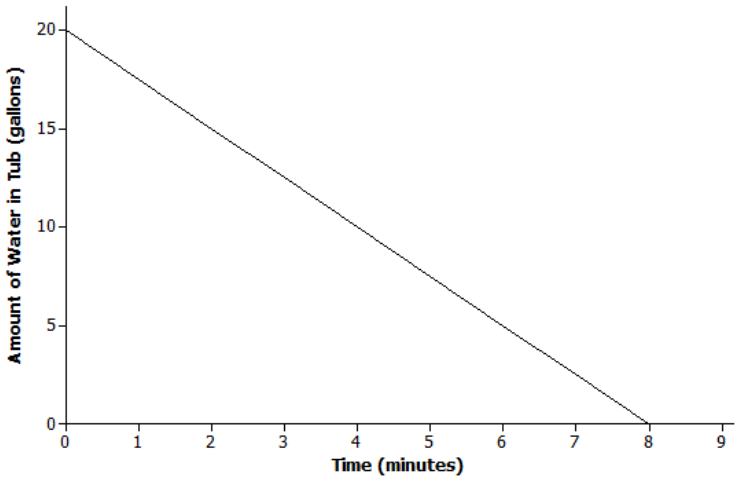
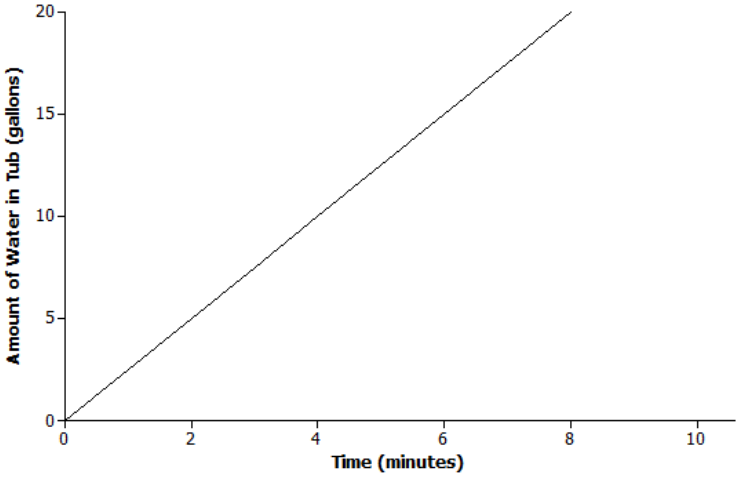
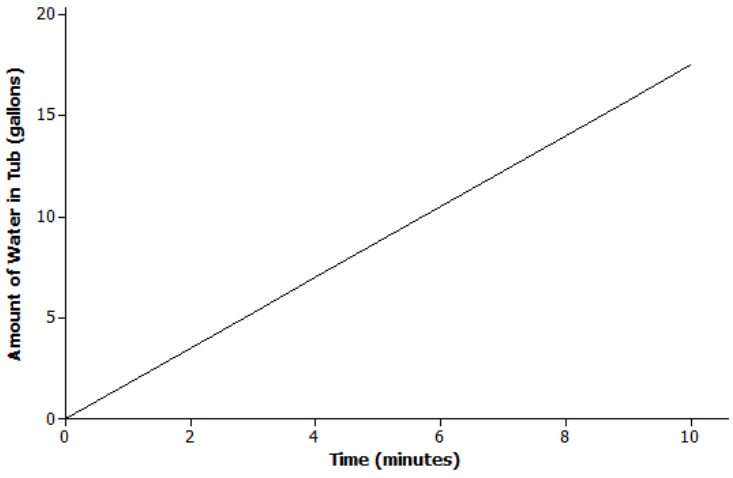
Exercises

1. Read through each of the scenarios and choose the graph of the function that best matches the situation. Explain the reason behind each choice.
   a. A bathtub is filled at a constant rate of 1.75 gallons per minute.
   b. A bathtub is drained at a constant rate of 2.5 gallons per minute.
   c. A bathtub contains 2.5 gallons of water.
   d. A bathtub is filled at a constant rate of 2.5 gallons per minute.



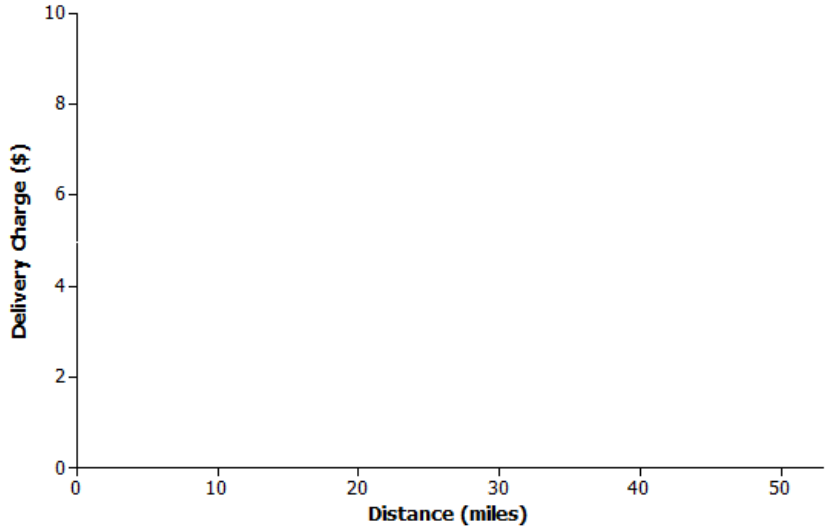
Scenario:

Explanation:

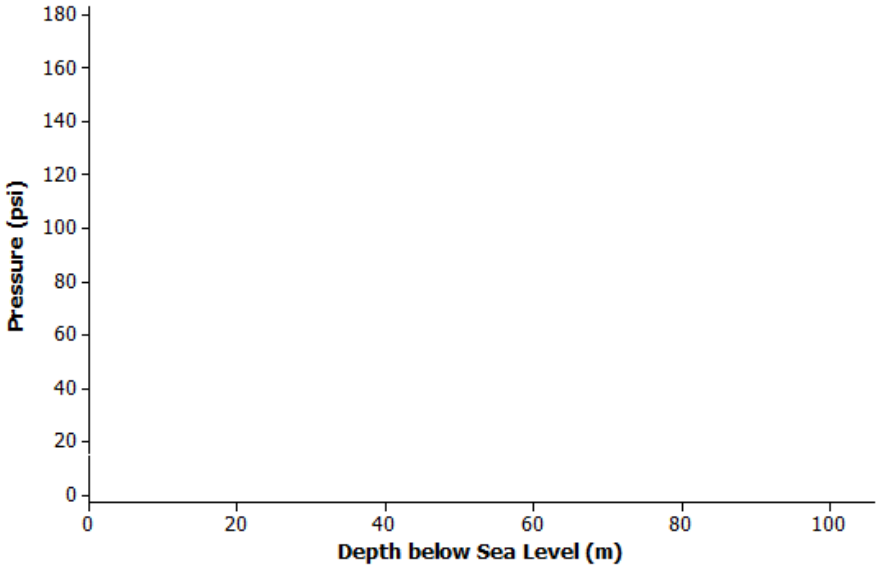
	**Scenario:** **Explanation:**
	**Scenario:** **Explanation:**
	**Scenario:** **Explanation:**

2. Read through each of the scenarios, and sketch a graph of a function that models the situation.

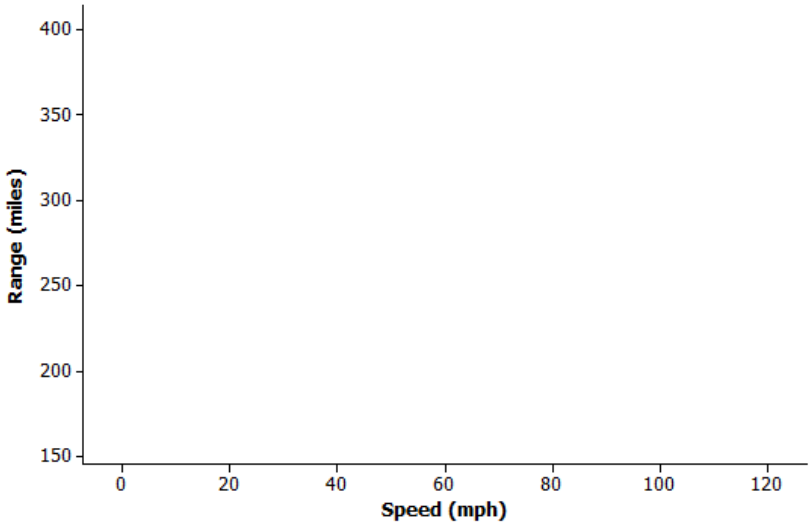
   a. A messenger service charges a flat rate of $\$4.95$ to deliver a package regardless of distance to the destination.



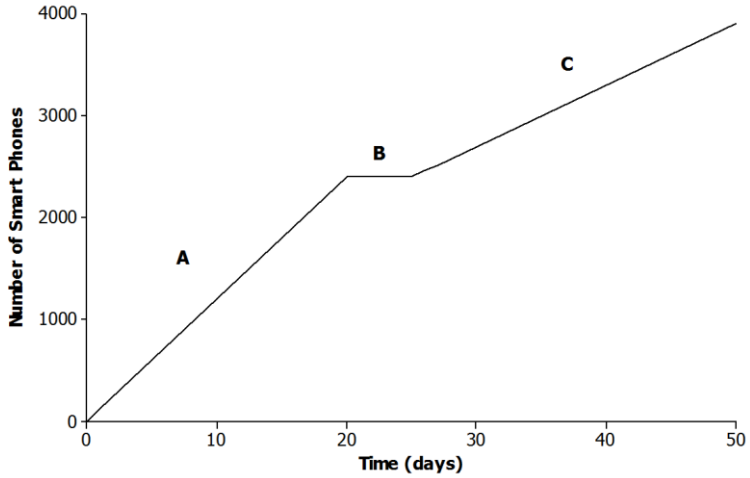
   b. At sea level, the air that surrounds us presses down on our bodies at 14.7 pounds per square inch (psi). For every 10 meters that you dive under water, the pressure increases by 14.7 psi.



c. The range (driving distance per charge) of an electric car varies based on the average speed the car is driven. The initial range of the electric car after a full charge is 400 miles. However, the range is reduced by 20 miles for every 10 mph increase in average speed the car is driven.



3. The graph below represents the total number of smart phones that are shipped to a retail store over the course of 50 days.



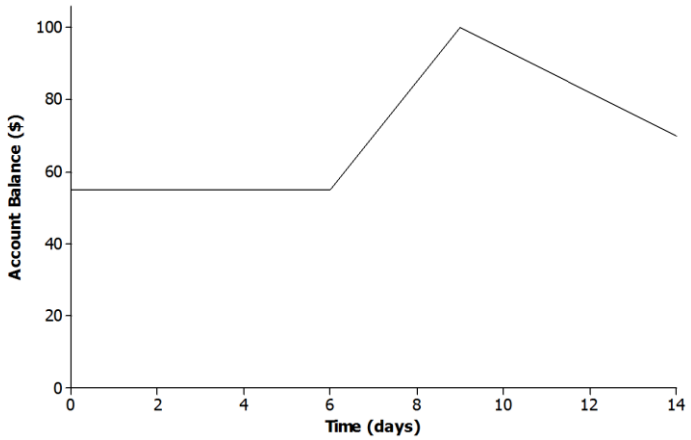
Match each part of the graph (A, B, and C) to its verbal description. Explain the reasoning behind your choice.

i. Half of the factory workers went on strike, and not enough smartphones were produced for normal shipments.

ii. The production schedule was normal, and smartphones were shipped to the retail store at a constant rate.

iii. A defective electronic chip was found, and the factory had to shut down; so, no smartphones were shipped.

4. The relationship between Jameson's account balance and time is modeled by the graph below.



a. Write a story that models the situation represented by the graph.

b. When is the function represented by the graph increasing? How does this relate to your story?

c. When is the function represented by the graph decreasing? How does this relate to your story?

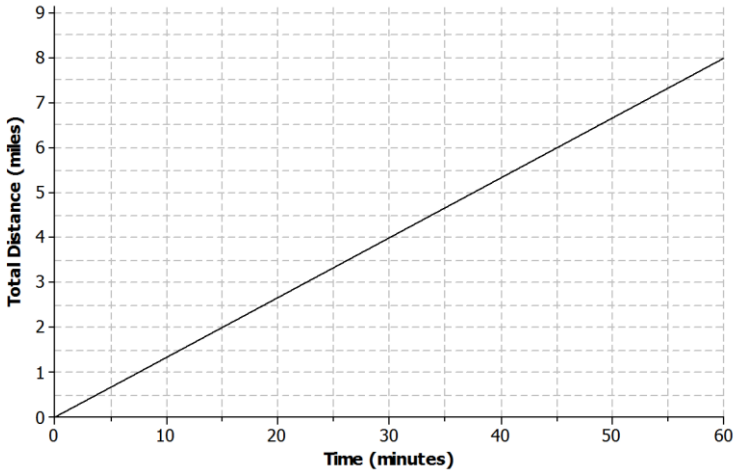
Lesson 5: Increasing and Decreasing Functions

Classwork

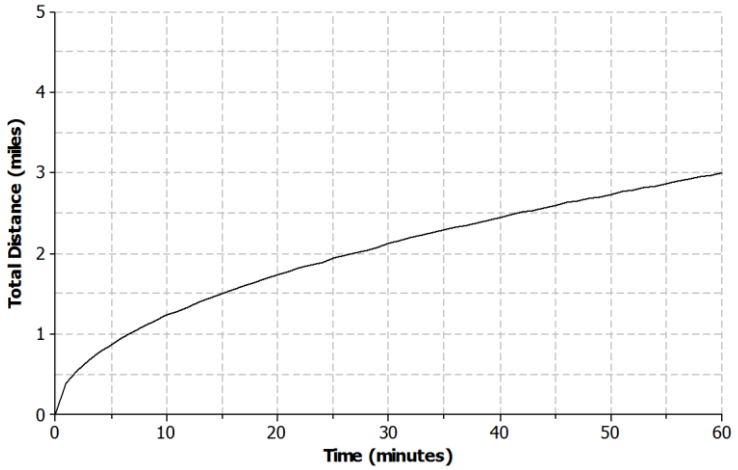
Example 1: Nonlinear Functions in the Real World

Not all real-world situations can be modeled by a linear function. There are times when a nonlinear function is needed to describe the relationship between two types of quantities. Compare the two scenarios:

a. Aleph is running at a constant rate on a flat paved road. The graph below represents the total distance he covers with respect to time.



b. Shannon is running on a flat, rocky trail that eventually rises up a steep mountain. The graph below represents the total distance she covers with respect to time.



Exercises 1–2

1. In your own words, describe what is happening as Aleph is running during the following intervals of time.
   a. 0 to 15 minutes

   b. 15 to 30 minutes

   c. 30 to 45 minutes

   d. 45 to 60 minutes

2. In your own words, describe what is happening as Shannon is running during the following intervals of time.
   a. 0 to 15 minutes

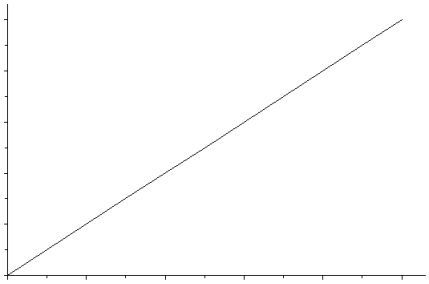
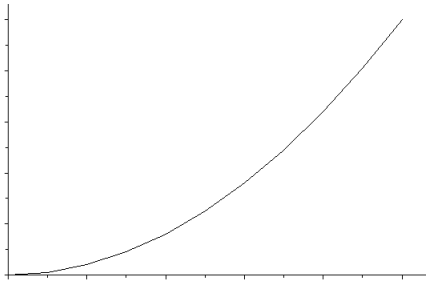
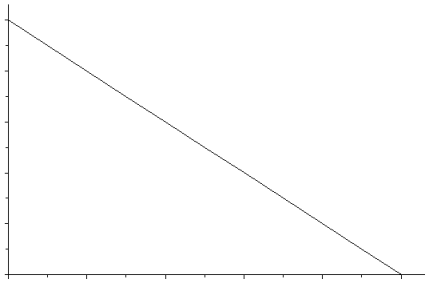
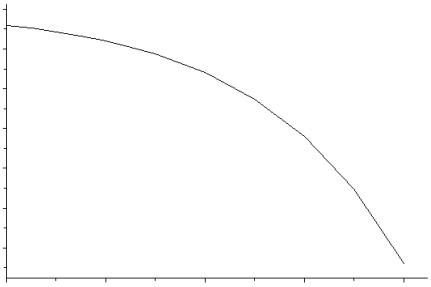
   b. 15 to 30 minutes

   c. 30 to 45 minutes

   d. 45 to 60 minutes

Example 2: Increasing and Decreasing Functions

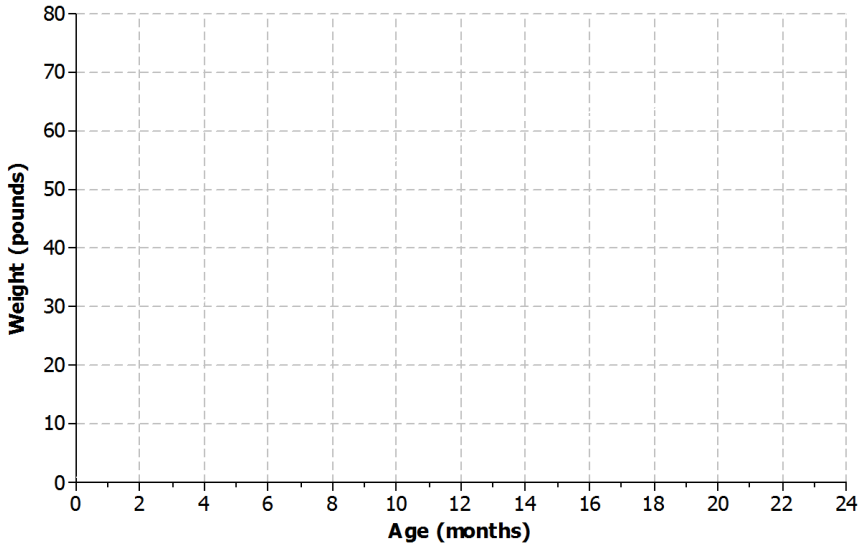
The rate of change of a function can provide useful information about the relationship between two quantities. A linear function has a constant rate of change. A nonlinear function has a variable rate of change.

<table>
<tr><th>Linear Functions</th><th>Nonlinear Functions</th></tr>
<tr><td>Linear function increasing at a constant rate
  </td><td>Nonlinear function increasing at a variable rate
  </td></tr>
<tr><td>Linear function decreasing at a constant rate
  </td><td>Nonlinear function decreasing at a variable rate
  </td></tr>
<tr><td>Linear function with a constant rate
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>7</td></tr><tr><td>1</td><td>10</td></tr><tr><td>2</td><td>13</td></tr><tr><td>3</td><td>16</td></tr><tr><td>4</td><td>19</td></tr></table></td><td>Nonlinear function with a variable rate
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>8</td></tr><tr><td>4</td><td>16</td></tr></table></td></tr>
</table>

Exercises 3–5

3. Different breeds of dogs have different growth rates. A large breed dog typically experiences a rapid growth rate from birth to age 6 months. At that point, the growth rate begins to slow down until the dog reaches full growth around 2 years of age.

   a. Sketch a graph that represents the weight of a large breed dog from birth to 2 years of age.

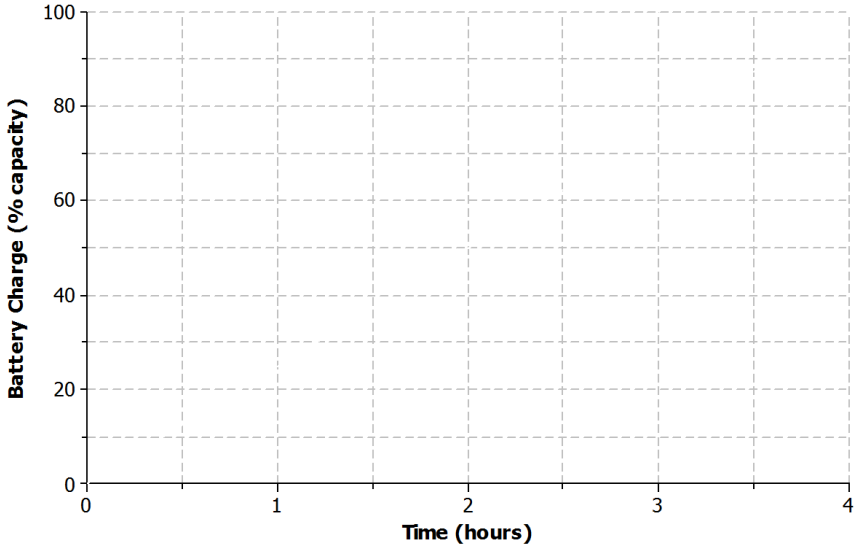


   b. Is the function represented by the graph linear or nonlinear? Explain.

   c. Is the function represented by the graph increasing or decreasing? Explain.

4. Nikka took her laptop to school and drained the battery while typing a research paper. When she returned home, Nikka connected her laptop to a power source, and the battery recharged at a constant rate.

   a. Sketch a graph that represents the battery charge with respect to time.

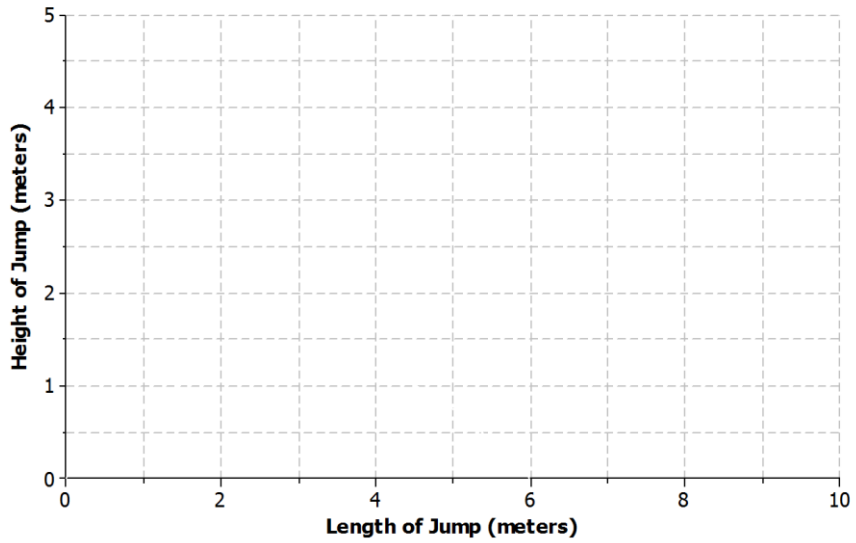


   b. Is the function represented by the graph linear or nonlinear? Explain.

   c. Is the function represented by the graph increasing or decreasing? Explain.

5. The long jump is a track and field event where an athlete attempts to leap as far as possible from a given point. Mike Powell of the United States set the long jump world record of 8.95 meters (29.4 feet) during the 1991 World Championships in Tokyo, Japan.

   a. Sketch a graph that represents the path of a high school athlete attempting the long jump.

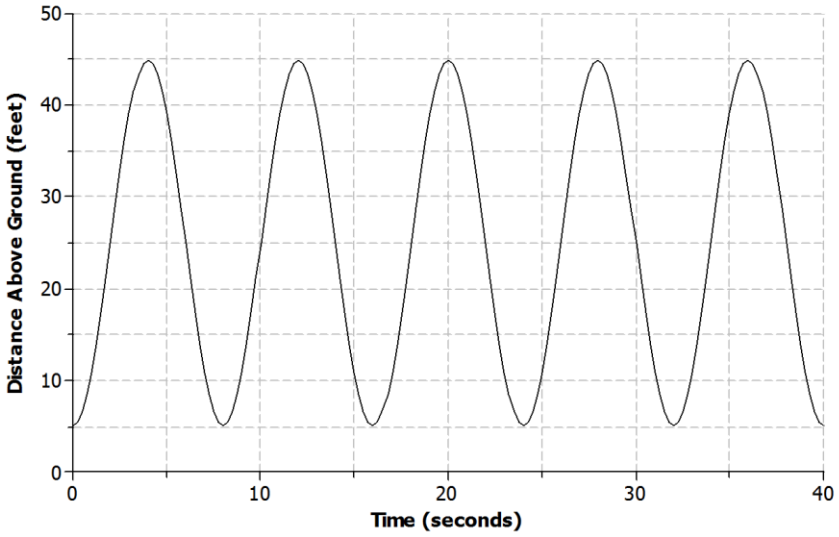


   b. Is the function represented by the graph linear or nonlinear? Explain.

   c. Is the function represented by the graph increasing or decreasing? Explain.

Example 3: Ferris Wheel

Lamar and his sister are riding a Ferris wheel at a state fair. Using their watches, they find that it take 8 seconds for the Ferris wheel to make a complete revolution. The graph below represents Lamar and his sister's distance above the ground with respect to time.



Exercises 6–9

6. Use the graph from Example 3 to answer the following questions.

   a. Is the function represented by the graph linear or nonlinear?

   b. Where is the function increasing? What does this mean within the context of the problem?

   c. Where is the function decreasing? What does this mean within the context of the problem?

7. How high above the ground is the platform for passengers to get on the Ferris wheel? Explain your reasoning.

8. Based on the graph, how many revolutions does the Ferris wheel complete during the 40 second time interval? Explain your reasoning.

9. What is the diameter of the Ferris wheel? Explain your reasoning.

Lesson 6: Scatter Plots

Classwork

Example 1

A bivariate data set consists of observations on two variables. For example, you might collect data on 13 different car models. Each observation in the data set would consist of an (x, y) pair.

x = weight (in pounds, rounded to the nearest 50 pounds)

and

y = fuel efficiency (in miles per gallon, mpg)

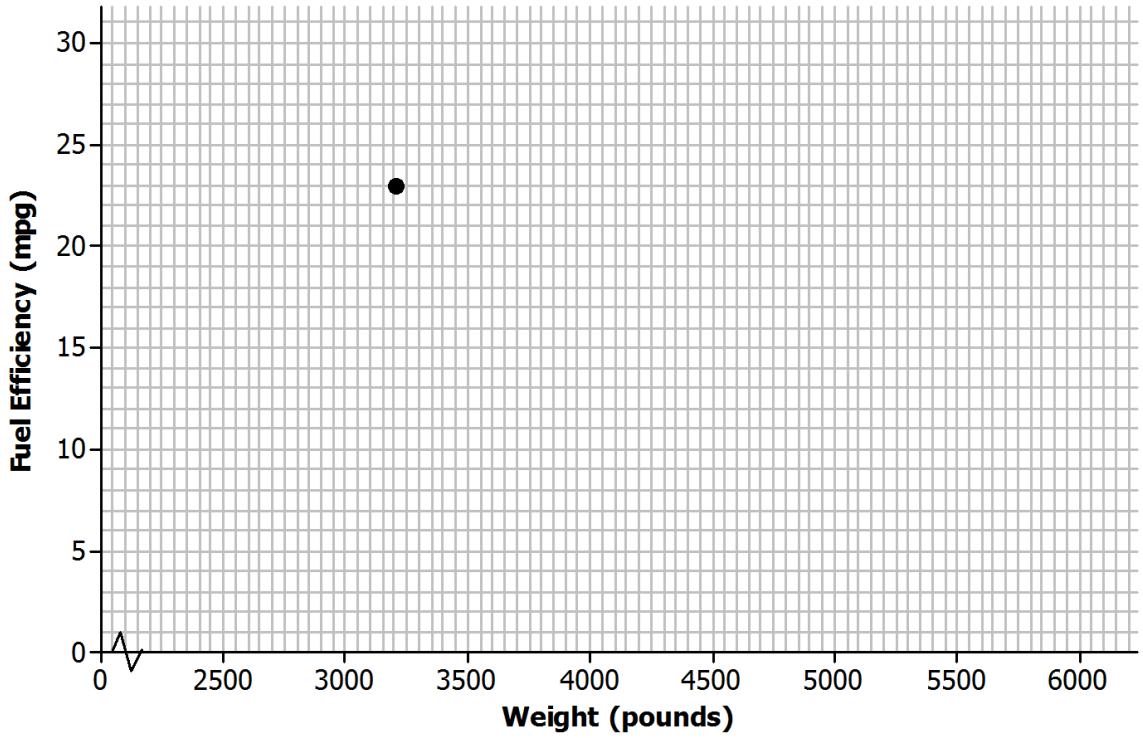
The table below shows the weight and fuel efficiency for 13 car models with automatic transmissions manufactured in 2009 by Chevrolet.

Model	Weight (pounds)	Fuel Efficiency (mpg)
1	3,200	23
2	2,550	28
3	4,050	19
4	4,050	20
5	3,750	20
6	3,550	22
7	3,550	19
8	3,500	25
9	4,600	16
10	5,250	12
11	5,600	16
12	4,500	16
13	4,800	15

Exercises 1–8

1. In the table above, the observation corresponding to Model 1 is $(3200, 23)$. What is the fuel efficiency of this car? What is the weight of this car?

2. Add the points corresponding to the other 12 observations to the scatter plot.



3. Do you notice a pattern in the scatter plot? What does this imply about the relationship between weight (x) and fuel efficiency (y)?

Is there a relationship between price and the quality of athletic shoes? The data in the table below are from the Consumer Reports website.

x = price (in dollars)

and

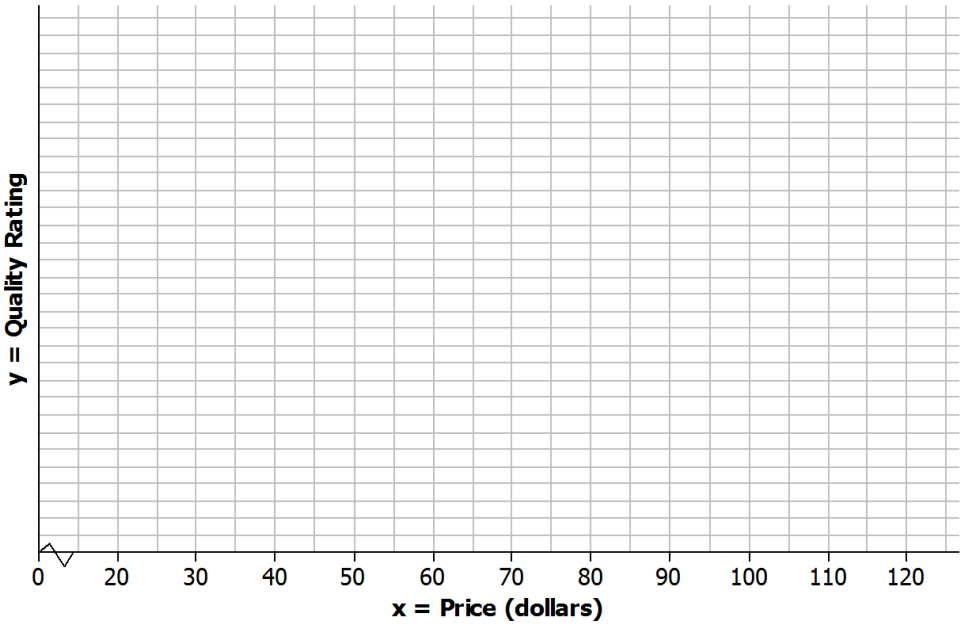
y = Consumer Reports quality rating

The quality rating is on a scale of 0 to 100, with 100 being the highest quality.

Shoe	Price (dollars)	Quality Rating
1	65	71
2	45	70
3	45	62
4	80	59
5	110	58
6	110	57
7	30	56
8	80	52
9	110	51
10	70	51

4. One observation in the data set is $(110, 57)$. What does this ordered pair represent in terms of cost and quality?

5. To construct a scatter plot of these data, you need to start by thinking about appropriate scales for the axes of the scatter plot. The prices in the data set range from \$30 to \$110, so one reasonable choice for the scale of the x-axis would range from \$20 to \$120, as shown below. What would be a reasonable



choice for a scale for the y-axis?

6. Add a scale to the y-axis. Then, use these axes to construct a scatter plot of the data.

7. Do you see any pattern in the scatter plot indicating that there is a relationship between price and quality rating for athletic shoes?

8. Some people think that if shoes have a high price, they must be of high quality. How would you respond?

Example 2: Statistical Relationships

A pattern in a scatter plot indicates that the values of one variable tend to vary in a predictable way as the values of the other variable change. This is called a *statistical relationship*. In the fuel efficiency and car weight example, fuel efficiency tended to decrease as car weight increased.

This is useful information, but be careful not to jump to the conclusion that increasing the weight of a car *causes* the fuel efficiency to go down. There may be some other explanation for this. For example, heavier cars may also have bigger engines, and bigger engines may be less efficient. You cannot conclude that changes to one variable *cause* changes in the other variable just because there is a statistical relationship in a scatter plot.

Exercises 9–10

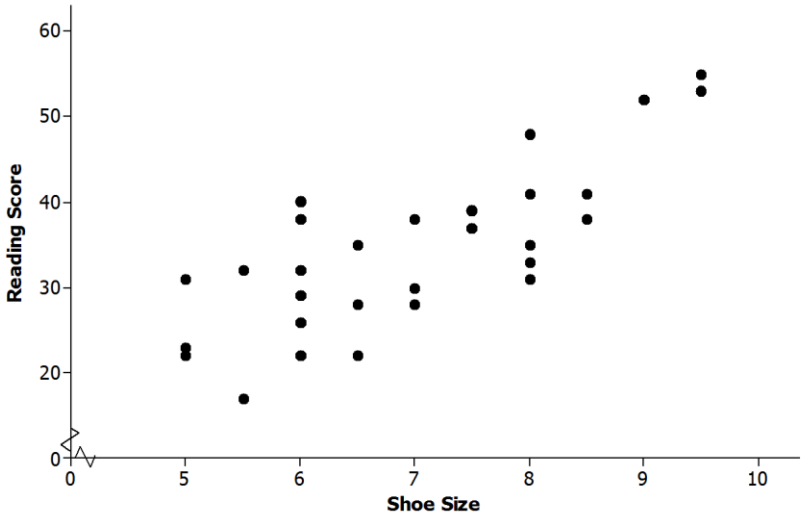
9. Data were collected on

x = shoe size

and

y = score on a reading-ability test

for 30 elementary school students. The scatter plot of these data is shown below. Does there appear to be a statistical relationship between shoe size and score on the reading test?



10. Explain why it is not reasonable to conclude that having big feet causes a high reading score. Can you think of a different explanation for why you might see a pattern like this?

Lesson 7: Patterns in Scatter Plots

Classwork

Example 1

In the previous lesson, you learned that scatterplots show trends in bivariate data.

When you look at a scatter plot, you should ask yourself the following questions:

a. Does it look like there is a relationship between the two variables used to make the scatter plot?

b. If there is a relationship, does it appear to be linear?

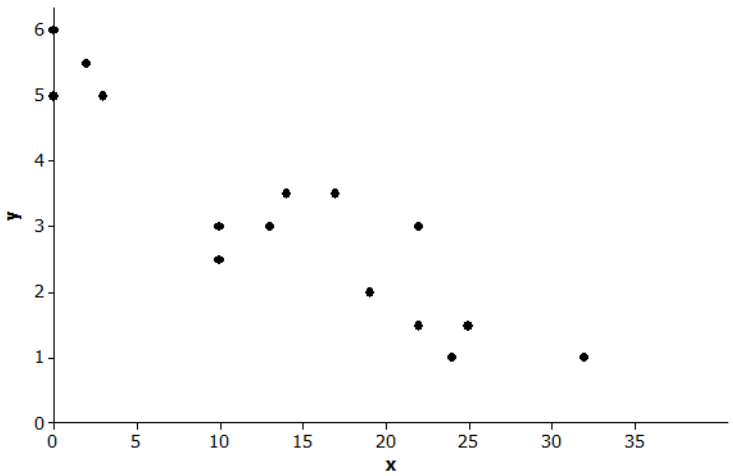
c. If the relationship appears to be linear, is the relationship a positive linear relationship or a negative linear relationship?

To answer the first question, look for patterns in the scatter plot. Does there appear to be a general pattern to the points in the scatter plot, or do the points look as if they are scattered at random? If you see a pattern, you can answer the second question by thinking about whether the pattern would be well-described by a line. Answering the third question requires you to distinguish between a positive linear relationship and a negative linear relationship. A positive linear relationship is one that is described by a line with a positive slope. A negative linear relationship is one that is described by a line with a negative slope.

Exercises 1–9

Take a look at the following five scatter plots. Answer the three questions above for each scatter plot.

1. Scatter Plot 1



Is there a relationship?

If there is a relationship, does it appear to be linear?

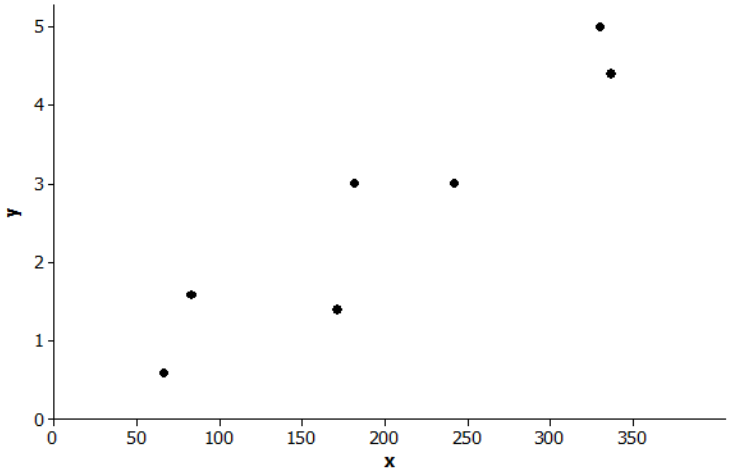
If the relationship appears to be linear, is it a positive or negative linear relationship?

2. Scatter Plot 2

Is there a relationship?

If there is a relationship, does it appear to be linear?

If the relationship appears to be linear, is it a positive or negative linear relationship?

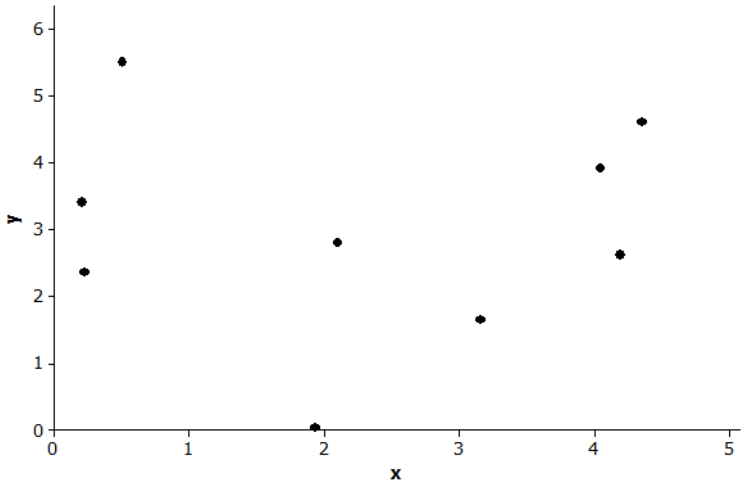


3. Scatter Plot 3

Is there a relationship?

If there is a relationship, does it appear to be linear?

If the relationship appears to be linear, is it a positive or negative linear relationship?

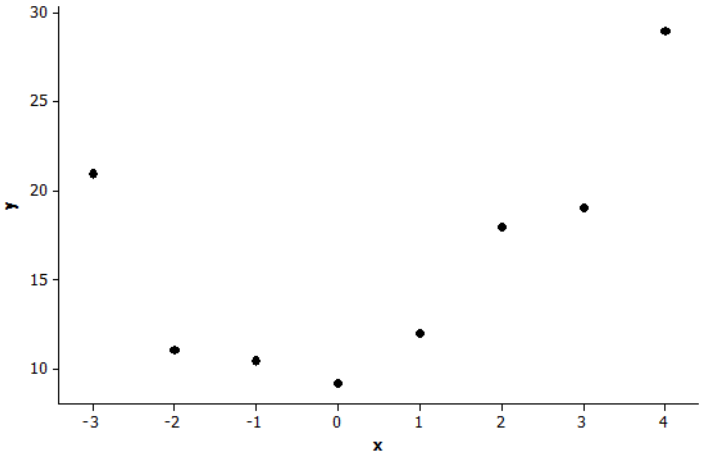


4. Scatter Plot 4

Is there a relationship?

If there is a relationship, does it appear to be linear?

If the relationship appears to be linear, is it a positive or negative linear relationship?

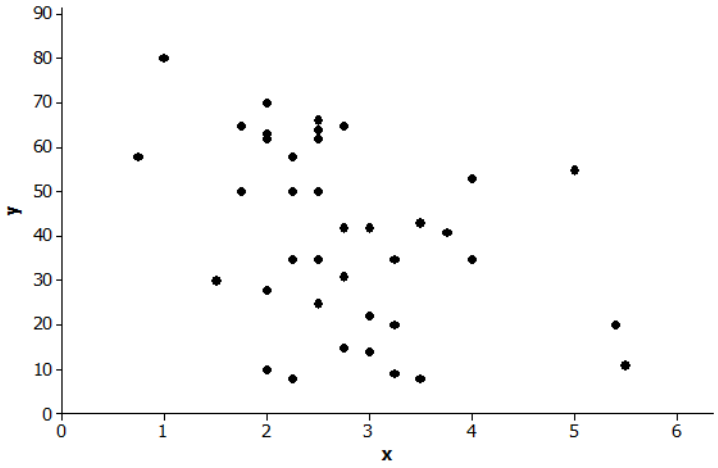


5. Scatter Plot 5

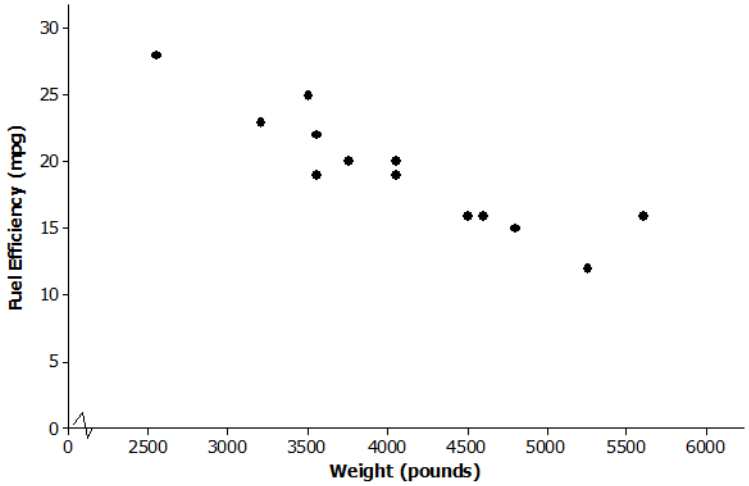
Is there a relationship?

If there is a relationship, does it appear to be linear?

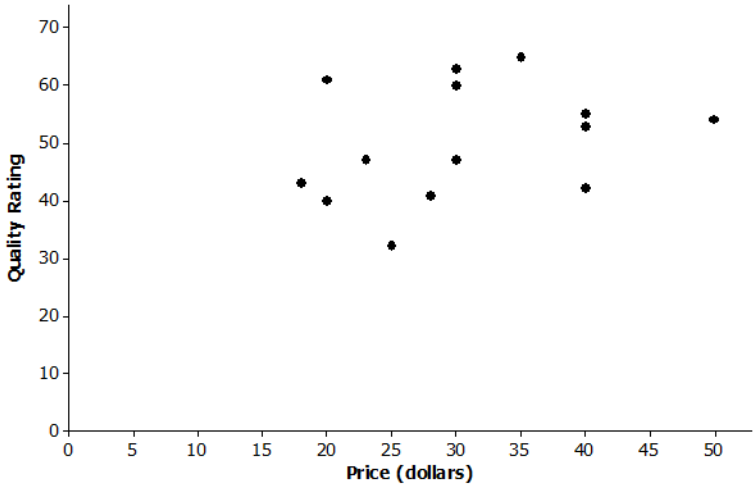
If the relationship appears to be linear, is it a positive or negative linear relationship?



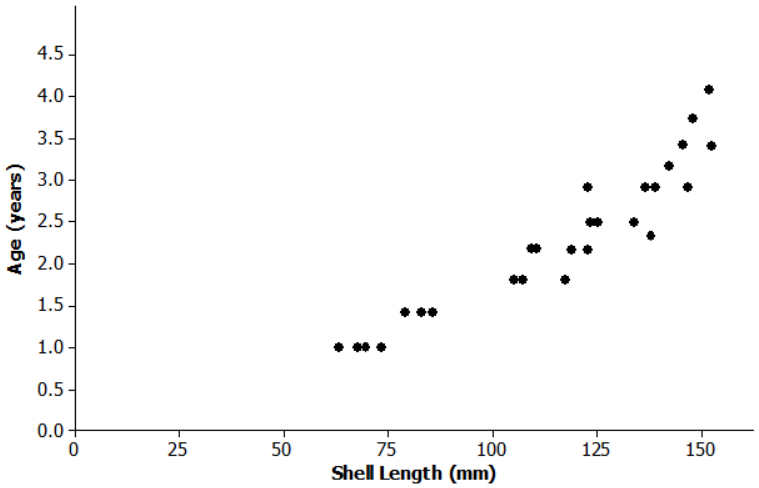
6. Below is a scatter plot of data on weight in pounds (x) and fuel efficiency in miles per gallon (y) for 13 cars. Using the questions at the beginning of this lesson as a guide, write a few sentences describing any possible relationship between x and y.



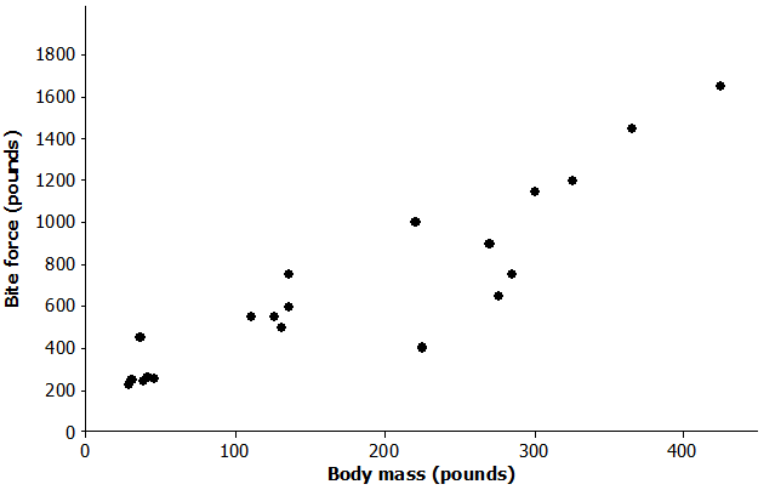
7. Below is a scatter plot of data on price in dollars (x) and quality rating (y) for 14 bike helmets. Using the questions at the beginning of this lesson as a guide, write a few sentences describing any possible relationship between x and y.



8. Below is a scatter plot of data on shell length in millimeters (x) and age in years (y) for 27 lobsters of known age. Using the questions at the beginning of this lesson as a guide, write a few sentences describing any possible relationship between x and y.



9. Below is a scatter plot of data from crocodiles on body mass in pounds (x) and bite force in pounds (y). Using the questions at the beginning of this lesson as a guide, write a few sentences describing any possible relationship between x and y.

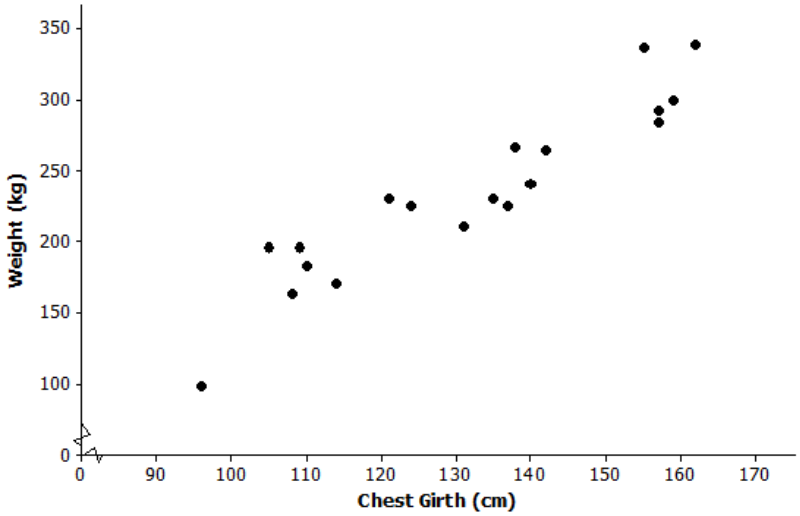


Example 2: Clusters and Outliers

In addition to looking for a general pattern in a scatter plot, you should also look for other interesting features that might help you understand the relationship between two variables. Two things to watch for are as follows:

- **CLUSTERS:** Usually the points in a scatter plot form a single cloud of points, but sometimes the points may form two or more distinct clouds of points. These clouds are called *clusters*. Investigating these clusters may tell you something useful about the data.
- **OUTLIERS:** An *outlier* is an unusual point in a scatter plot that does not seem to fit the general pattern or that is far away from the other points in the scatter plot.

The scatter plot below was constructed using data from a study of Rocky Mountain elk ("Estimating Elk Weight from Chest Girth," *Wildlife Society Bulletin*, 1996). The variables studied were chest girth in centimeter (x) and weight in kilogram (y).



Exercises 10–12

10. Do you notice any point in the scatter plot of elk weight versus chest girth that might be described as an outlier? If so, which one?

11. If you identified an outlier in Exercise 10, write a sentence describing how this data observation differs from the others in the data set.

12. Do you notice any clusters in the scatter plot? If so, how would you distinguish between the clusters in terms of chest girth? Can you think of a reason these clusters might have occurred?

Lesson 8: Informally Fitting a Line

Classwork

Example 1: Housing Costs

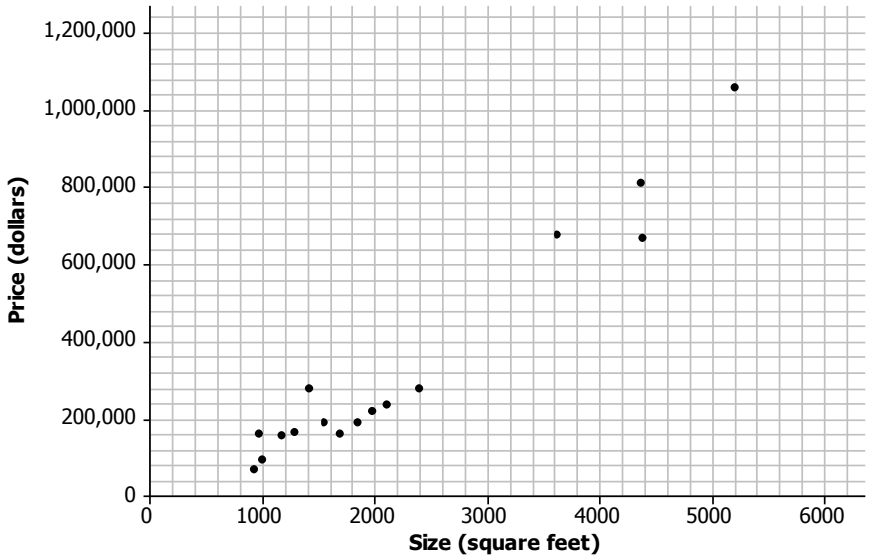
Let's look at some data from one Midwestern city that indicates the sizes and sale prices of various houses sold in this city.

Size (square feet)	Price (dollars)
5,232	1,050,000
1,875	179,900
1,031	84,900
1,437	269,900
4,400	799,900
2,000	209,900
2,132	224,900
1,591	179,900

Size (square feet)	Price (dollars)
1,196	144,900
1,719	149,900
956	59,900
991	149,900
1,312	154,900
4,417	659,999
3,664	669,000
2,421	269,900

Data Source: http://www.trulia.com/for_sale/Milwaukee,WI/5_p

A scatter plot of the data is given below.



Exercises 1–6

1. What can you tell about the price of large homes compared to the price of small homes from the table?

2. Use the scatter plot to answer the following questions.
   a. Does the scatter plot seem to support the statement that larger houses tend to cost more? Explain your thinking.

   b. What is the cost of the most expensive house, and where is that point on the scatter plot?

   c. Some people might consider a given amount of money and then predict what size house they could buy. Others might consider what size house they want and then predict how much it would cost. How would you use the above scatter plot?

   d. Estimate the cost of a $3{,}000$ square foot house.

   e. Do you think a line would provide a reasonable way to describe how price and size are related? How could you use a line to predict the price of a house if you are given its size?

3. Draw a line in the plot that you think would fit the trend in the data.
4. Use your line to answer the following questions:
   a. What is your prediction of the price of a $3{,}000$ square foot house?

   b. What is the prediction of the price of a $1{,}500$ square foot house?

5. Consider the following general strategies students use for drawing a line. Do you think they represent a good strategy for drawing a line that will fit the data? Explain why or why not, or draw a line for the scatter plot using the strategy that would indicate why it is or why it is not a good strategy.
   a. Laure thought she might draw her line using the very first point (farthest to the left) and the very last point (farthest to the right) in the scatter plot.

   b. Phil wants to be sure that he has the same number of points above and below the line.

   c. Sandie thought she might try to get a line that had the most points right on it.

   d. Maree decided to get her line as close to as many of the points as possible.

6. Based on the strategies discussed in Exercise 5, would you change how you draw a line through the points? Explain your answer.

Example 2: Deep Water

Does the current in the water go faster or slower when the water is shallow? The data on the depth and speed of the Columbia River at various locations in Washington state listed below can help you think about the answer.

Depth and Velocity in the Columbia River, Washington State

Depth (feet)	Velocity (feet/second)
0.7	1.55
2.0	1.11
2.6	1.42
3.3	1.39
4.6	1.39
5.9	1.14
7.3	0.91
8.6	0.59
9.9	0.59
10.6	0.41
11.2	0.22

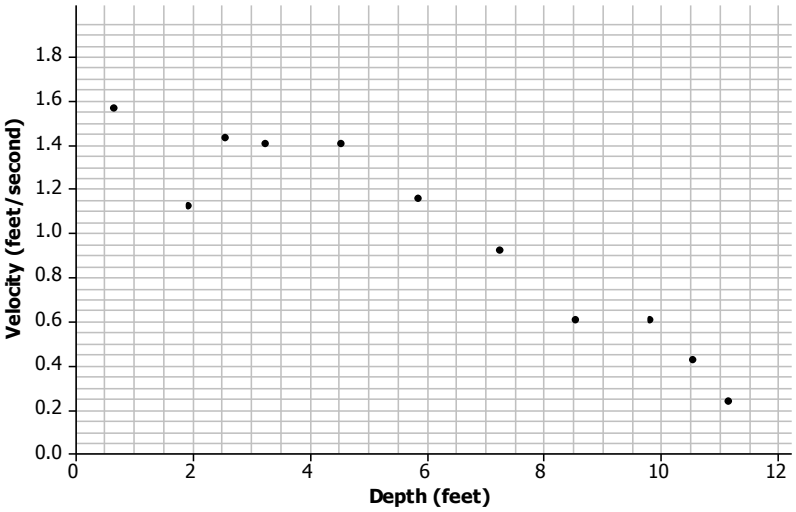
Data Source: www.seattlecentral.edu/qelp/sets/011/011.html

a. What can you tell about the relationship between the depth and velocity by looking at the numbers in the table?

b. If you were to make a scatter plot of the data, which variable would you put on the horizontal axis and why?

Exercises 7–9

7. A scatter plot of the Columbia River data is shown below.



a. Choose a data point in the scatter plot and describe what it means in terms of the context.

b. Based on the scatter plot, describe the relationship between velocity and depth.

c. How would you explain the relationship between the velocity and depth of the water?

d. If the river is two feet deep at a certain spot, how fast do you think the current would be? Explain your reasoning.

8. Consider the following questions:
   a. If you draw a line to represent the trend in the plot, will it make it easier to predict the velocity of the water if you know the depth? Why or why not?

   b. Draw a line that you think does a reasonable job of modeling the trend on the scatter plot above. Use the line to predict the velocity when the water is 8 feet deep.

9. Use the line to predict the velocity for a depth of 8.6 feet. How far off was your prediction from the actual observed velocity for the location that had a depth of 8.6 feet?

Lesson 9: Determining the Equation of a Line Fit to Data

Classwork

Example 1: Crocodiles and Alligators

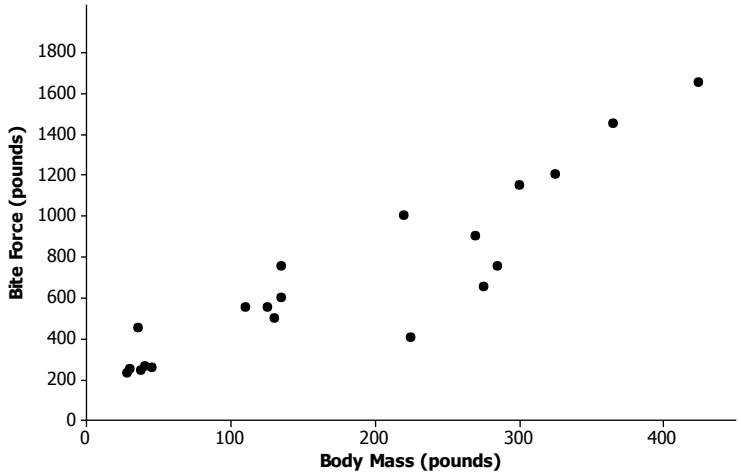
Scientists are interested in finding out how different species adapt to finding food sources. One group studied crocodilian to find out how their bite force was related to body mass and diet. The table below displays the information they collected on body mass (in pounds) and bite force (in pounds).

Crocodilian Biting

Species	Body Mass (pounds)	Bite Force (pounds)
Dwarf Crocodile	35	450
Crocodile F	40	260
Alligator A	30	250
Caiman A	28	230
Caiman B	37	240
Caiman C	45	255
Crocodile A	110	550
Nile Crocodile	275	650
Crocodile B	130	500
Crocodile C	135	600
Crocodile D	135	750
Caiman D	125	550
Indian Gharial Crocodile	225	400
Crocodile G	220	1,000
American Crocodile	270	900
Crocodile D	285	750
Crocodile E	425	1,650
American Alligator	300	1,150
Alligator B	325	1,200
Alligator C	365	1,450

Data Source: PLoS One Greg Erickson biomechanics, Florida State University

As you learned in the previous lesson, it is a good idea to begin by looking at what a scatter plot tells you about the data. The scatter plot below displays the data on body mass and bite force for the crocodilian in the study.



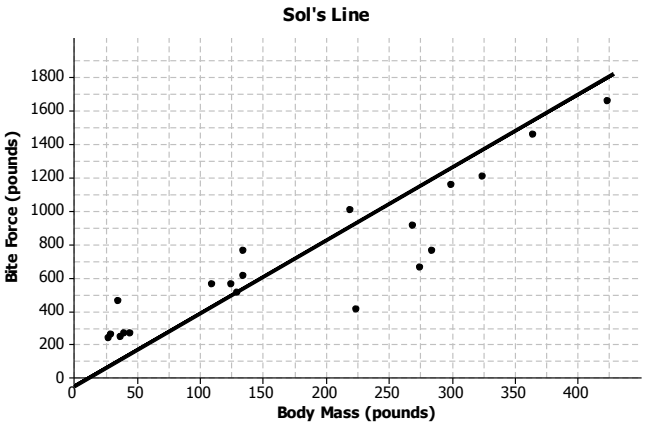
Exercises 1–6

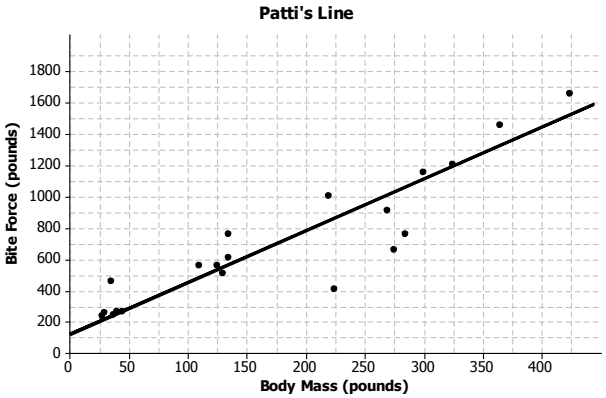
1. Describe the relationship between body mass and bite force for the crocodilian shown in the scatter plot.

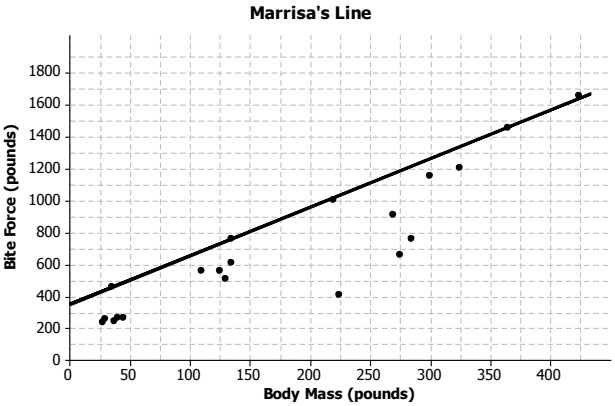
2. Draw a line to represent the trend in the data. Comment on what you considered in drawing your line.

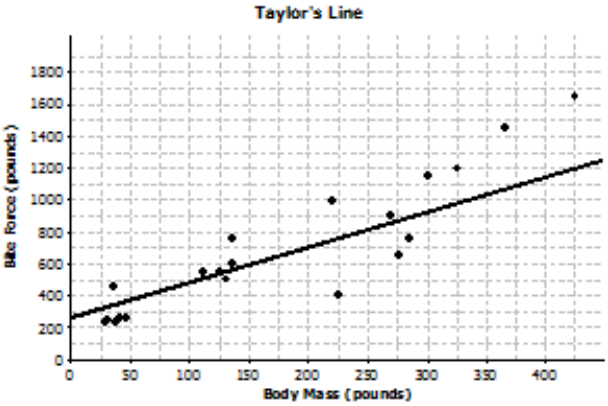
3. Based on your line, predict the bite force for a crocodilian that weighs 220 pounds. How does this prediction compare to the actual bite force of the 220-pound crocodilian in the data set?

4. Several students decided to draw lines to represent the trend in the data. Consider the lines drawn by Sol, Patti, Marrisa, and Taylor, which are shown below.









For each student, indicate whether or not you think the line would be a good line to use to make predictions. Explain your thinking.

a. Sol's line

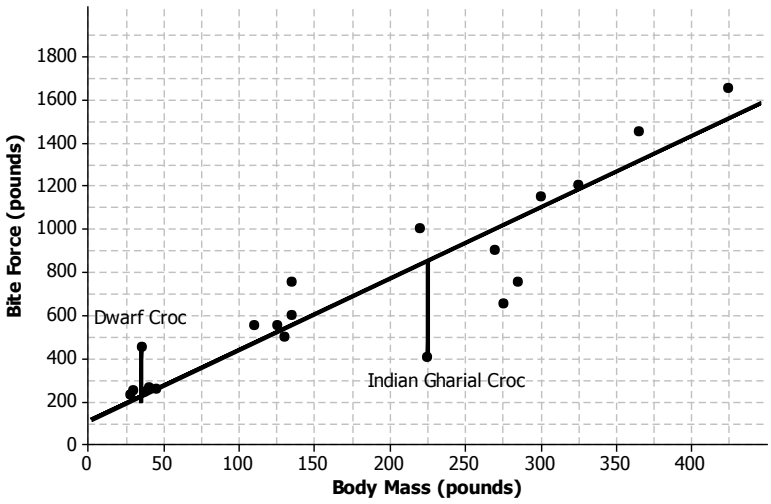
b. Patti's line

c. Marrisa's line

d. Taylor's line

5. What is the equation of your line? Show the steps you used to determine your line. Based on your equation, what is your prediction for the bite force of a crocodilian weighing 200 pounds?

6. Patti drew vertical line segments from two points to the line in her scatter plot. The first point she selected was for a Dwarf Crocodile. The second point she selected was for an Indian Gharial Crocodile.

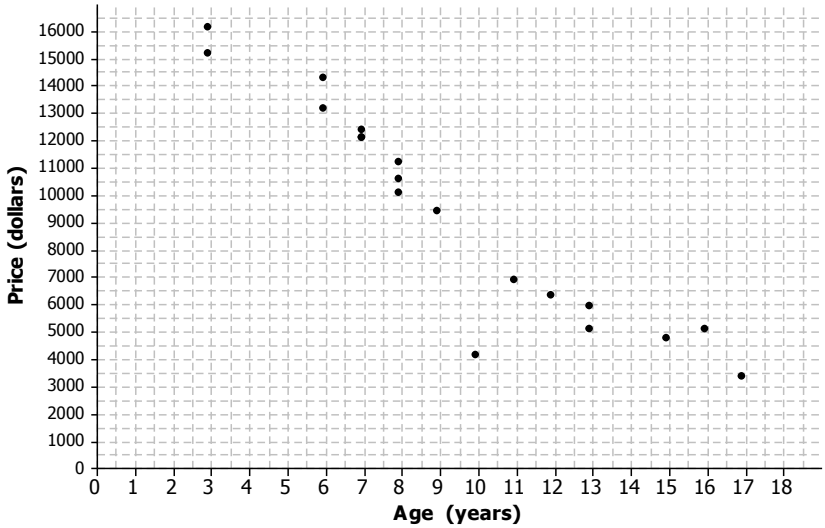


a. Would Patti's line have resulted in a predicted bite force that was closer to the actual bite force for the Dwarf Crocodile or for the Indian Gharial Crocodile? What aspect of the scatter plot supports your answer?

b. Would it be preferable to describe the trend in a scatter plot using a line that makes the differences in the actual and predicted values large or small? Explain your answer.

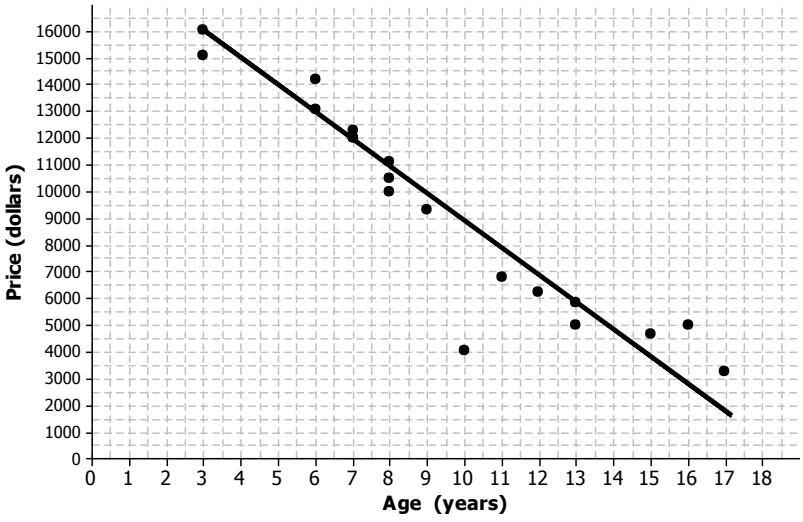
Exercise 7: Used Cars

7. The plot below shows the age (in years) and price (in dollars) of used Honda Civic cars that were advertised in a local newspaper.



a. Based on the scatter plot above, describe the relationship between the age and price of the used cars.

b. Nora drew a line she thought was close to many of the points and found the equation of the line. She used the points $(13, 6000)$ and $(7, 12000)$ on her line to find the equation. Explain why those points made finding the equation easy.



c. Find the equation of Nora's line for predicting the price of a used car given its age. Summarize the trend described by this equation.

d. Based on the line, for which car in the data set would the predicted value based on the line be farthest from the actual value? How can you tell?

e. What does the equation predict for the cost of a 10-year-old car? How close was the prediction using the line to the actual cost of the 10-year-old car in the data set? Given the context of the data set, do you think the difference between the predicted price and the actual price is large or small?

f. Is \$5,000 typical of the differences between predicted prices and actual prices for the cars in this data set? Justify your answer.

Lesson 13: Summarizing Bivariate Categorical Data in a Two-Way Table

Classwork

Exercises 1–8

On an upcoming field day at school, the principal wants to provide ice cream during lunch. She will offer three flavors: chocolate, strawberry, and vanilla. She selected your class to complete a survey to help her determine how much of each flavor to buy.

1. Answer the following question. Wait for your teacher to count how many students selected each flavor. Then, record the class totals for each flavor in the chart below.

   "Which of the following three ice cream flavors is your favorite: chocolate, strawberry, or vanilla?"

Ice Cream Flavor	Chocolate	Strawberry	Vanilla	Total
Number of Students				

2. Which ice cream flavor do most students prefer?

3. Which ice cream flavor do the fewest students prefer?

4. What percentage of students preferred each flavor? Round to the nearest tenth of a percent.

5. Do the numbers in the chart above summarize data on a categorical variable or a numerical variable?

6. Do the students in your class represent a random sample of all students in your school? Why or why not? Discuss this with your neighbor.

7. Is your class representative of all the other classes at your school? Why or why not? Discuss this with your neighbor.

8. Do you think the principal will get an accurate estimate of the proportion of students that prefer each ice cream flavor for the whole school using only your class? Why or why not? Discuss this with your neighbor.

Example 1

Students in a different class were asked the same question about their favorite ice cream flavor. The table below shows the ice cream flavors and the number of students who chose each flavor for that particular class. This table is called a one-way frequency table because it shows the counts of a univariate categorical variable.

This is the univariate categorical variable. →

These are the counts for each category. →

Ice Cream Flavor	Chocolate	Strawberry	Vanilla	Total
Number of Students	11	4	10	25

We compute the relative frequency for each ice cream flavor by dividing the count by the total number of observations.

$$\text{relative frequency} = \frac{\text{count for a category}}{\text{total number of observations}}$$

Since 11 out of 25 students answered chocolate, the relative frequency would be $\frac{11}{25} = 0.44$. This relative frequency shows that 44% of the class prefers chocolate ice cream. In other words, the relative frequency is the proportional value that each category is of the whole.

Exercises 9–10

Use the table for the preferred ice cream flavors from the class in Example 1 to answer the following questions.

9. What is the relative frequency for the category strawberry?

10. Write a sentence interpreting the relative frequency value in the context of strawberry ice cream preference.

Example 2

The principal also wondered if boys and girls have different favorite ice cream flavors. She decided to redo the survey by taking a random sample of students from the school and recording both their favorite ice cream flavor and their gender. She asked the following two questions:

- "Which of the following ice cream flavors is your favorite: chocolate, strawberry, or vanilla?"
- "What is your gender: male or female?"

The results of the survey are as follows:

- Of the 30 students who prefer chocolate ice cream, 22 are males.
- Of the 25 students who prefer strawberry ice cream, 15 are females.
- Of the 27 students who prefer vanilla ice cream, 13 are males.

The values of two variables, which were ice cream flavor and gender, were recorded in this survey. Since both of the variables are categorical, the data are bivariate categorical data.

Exercises 11–17

11. Can we display these data in a one-way frequency table? Why or why not?

12. Summarize the results of the second survey of favorite ice cream flavors in the following table:

		Favorite Ice Cream Flavor			
		Chocolate	Strawberry	Vanilla	Total
Gender	Male				
	Female				
	Total				

13. Calculate the relative frequencies for the table above and write them in the table.

		Favorite Ice Cream Flavor			
		Chocolate	Strawberry	Vanilla	Total
Gender	Male				
	Female				
	Total				

Use the relative frequency values in the table to answer the following questions:

14. What is the proportion of the students that prefer chocolate ice cream?

15. What is the proportion of students that are female and prefer vanilla ice cream?

16. Write a sentence explaining the meaning of the approximate relative frequency 0.55.

17. Write a sentence explaining the meaning of the approximate relative frequency 0.10.

Example 3

In the previous exercises, you used the total number of students to calculate relative frequencies. These relative frequencies were the proportion of the whole group who answered the survey a certain way. Sometimes we use row or column totals to calculate relative frequencies. We call these *row relative frequencies* or *column relative frequencies*.

Below is the two-way frequency table for your reference. To calculate "the proportion of male students that prefer chocolate ice cream," divide the 22 male students who preferred chocolate ice cream by the total of 45 male students. This proportion is $\frac{22}{45} = 0.49$. Notice that you used the row total to make this calculation. This is a row relative frequency.

		Favorite Ice Cream Flavor			
		Chocolate	Strawberry	Vanilla	Total
Gender	Male	22	10	13	45
	Female	8	15	14	37
	Total	30	25	27	82

Exercises 18–22

In Exercise 13, you used the total number of students to calculate relative frequencies. These relative frequencies were the proportion of the whole group who answered the survey a certain way.

18. Suppose you are interested in the proportion of male students that prefer chocolate ice cream. How is this value different from "the proportion of students that are male and prefer chocolate ice cream"? Discuss this with your neighbor.

19. Use the table provided in Example 3 to calculate the following relative frequencies.
    a. What proportion of students that prefer vanilla ice cream is female?

    b. What proportion of male students prefers strawberry ice cream? Write a sentence explaining the meaning of this proportion in context of this problem.

c. What proportion of female students prefers strawberry ice cream?

d. What proportion of students who prefer strawberry ice cream is female?

20. A student is selected at random from this school. What would you predict this student's favorite ice cream to be? Explain why you choose this flavor.

21. Suppose the randomly selected student is male. What would you predict his favorite flavor of ice cream to be? Explain why you choose this flavor.

22. Suppose the randomly selected student is female. What would you predict her favorite flavor of ice cream to be? Explain why you choose this flavor.

Lesson 14: Association Between Categorical Variables

Classwork

Example 1

Suppose a random group of people are surveyed about their use of smartphones. The results of the survey are summarized in the tables below.

Smartphone Use and Gender

	Use Smartphone	Do not Use Smartphone	Total
Male	30	10	40
Female	45	15	60
Total	75	25	100

Smartphone Use and Age

	Use Smartphone	Do not Use Smartphone	Total
Under 40 Years of Age	45	5	50
40 Years of Age or Older	30	20	50
Total	75	25	100

Example 2

Suppose a sample of 400 participants (teachers and students) was randomly selected from the middle schools and high schools in a large city. These participants responded to the question:

Which type of movie do you prefer to watch?

1. Action (*The Avengers*, *Man of Steel*, etc.)
2. Drama (*42 (The Jackie Robinson Story)*, *The Great Gatsby*, etc.)
3. Science-Fiction (*Star Trek Into Darkness*, *World War Z*, etc.)
4. Comedy (*Monsters University*, *Despicable Me 2*, etc.)

Movie preference and status (teacher/student) were recorded for each participant.

Exercises 1–7

1. Two variables were recorded. Are these variables categorical or numerical?

2. The results of the survey are summarized in the table below.

	Movie Preference				
	Action	Drama	Science-Fiction	Comedy	Total
Student	120	60	30	90	300
Teacher	40	20	10	30	100
Total	160	80	40	120	400

   a. What proportion of participants who are teachers prefer action movies?

   b. What proportion of participants who are teachers prefer drama movies?

   c. What proportion of participants who are teachers prefer science-fiction movies?

   d. What proportion of participants who are teachers prefer comedy movies?

The answers to Exercise 2 are called row relative frequencies. Notice that you divided each cell frequency in the teacher row by the row total for that row. Below is a blank relative frequency table.

Table of Row Relative Frequencies

	Movie Preference			
	Action	Drama	Science-Fiction	Comedy
Student				
Teacher	(a)	(b)	(c)	(d)

Write your answers from Exercise 2 in the indicated cells in the table above.

3. Find the row relative frequencies for the student row. Write your answers in the table above.
   a. What proportion of participants who are students prefers action movies?
   b. What proportion of participants who are students prefers drama movies?
   c. What proportion of participants who are students prefers science-fiction movies?
   d. What proportion of participants who are students prefers comedy movies?

4. Is a participant's status (i.e., teacher or student) related to what type of movie he or she would prefer to watch? Why or why not? Discuss this with your group.

5. What does it mean when we say that there is no association between two variables? Discuss this with your group.

6. Notice that the row relative frequencies for each movie type are the same for both the teacher and student rows. When this happens we say that the two variables, movie preference and status (student/teacher), are NOT associated. Another way of thinking about this is to say that knowing if a participant is a teacher (or a student) provides no information about his or her movie preference.
   What does it mean if row relative frequencies are not the same for all rows of a two-way table?

7. You can also evaluate whether two variables are associated by looking at column relative frequencies instead of row relative frequencies. A column relative frequency is a cell frequency divided by the corresponding column total. For example, the column relative frequency for the Student-Action cell is $\frac{120}{160} = 0.75$.

   a. Calculate the other column relative frequencies and write them in the table below.

Table of Row Relative Frequencies

	Movie Preference			
	Action	Drama	Science-Fiction	Comedy
Student				
Teacher				

   b. What do you notice about the column relative frequencies for the four columns?

   c. What would you conclude about association based on the column relative frequencies?

Example 3

In the survey described in Example 2, gender for each of the 400 participants was also recorded. Some results of the survey are given below:

- 160 participants preferred action movies.
- 80 participants preferred drama movies.
- 40 participants preferred science-fiction movies.
- 240 participants were females.
- 78 female participants preferred drama movies.
- 32 male participants preferred science-fiction movies.
- 60 female participants preferred action movies.

Exercises 8–15

Use the results from Example 3 to answer the following questions. Be sure to discuss these questions with your group members.

8. Complete the two-way frequency table that summarizes the data on movie preference and gender.

	Movie Preference				
	Action	Drama	Science-Fiction	Comedy	Total
Student					
Teacher					
Total					

9. What proportion of the participants is female?

10. If there were no association between gender and movie preference, should you expect more females than males or fewer females than males to prefer action movies? Explain.

11. Make a table of row relative frequencies of each movie type for the male row and the female row. Refer to Exercises 2–4 to review how to complete the table below.

	Movie Preference			
	Action	Drama	Science-Fiction	Comedy
Student				
Teacher				

Suppose that you randomly pick 1 of the 400 participants. Use the table of row relative frequencies above to answer the following questions.

12. If you had to predict what type of movie this person chose, what would you predict? Explain why you made this choice.

13. If you know that the randomly selected participant is female, would you predict that her favorite type of movie was action? If not, what would you predict and why?

14. If knowing the value of one of the variables provides information about the value of the other variable, then there is an association between the two variables.

    Is there an association between the variables gender and movie preference? Explain.

15. So what can be said when two variables are associated? Read the following sentences. Decide if the sentence is a correct statement based upon the survey data. If it is not correct, explain why not.

    a. More females than males participated in the survey.

    b. Males tend to prefer action and science-fiction moves.

    c. Being female causes one to prefer drama movies.